STRATEGIC IMPLICATIONS OF US FIGHTER FORCE REDUCTIONS: AIR-TO-AIR COMBAT MODELING USING LANCHESTER EQUATIONS

GRADUATE RESEARCH PAPER

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Abstract

Lanchester equations are used as the foundation for analysis of air superiority forces, mathematically addressing the impending shortage of the United States fighters; focusing on the role of advanced technology: stealth aircraft, air-to-air missiles, and the rapid proliferation of electronic attack capabilities. These factors are accounted for in determination of the attrition coefficients for heterogeneous fighter aircraft through a simplistic scoring methodology and compared to potential adversarial states. ARENA simulation is employed to determine minimal fighter requirements and expected blue force losses as a function of threat force size and capability.

Analysis concludes that the United States is incapable of fighting a forward deployed aerial battle against a numerically equal or superior force that employs advanced technology unless initial force strength is at least half the adversarial totals. It is recommended that the US leverage innovation and advance specific technological areas surrounding fighter force lethality and survivability to address the deficiency in aircraft numbers for the foreseeable future.

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STRATEGIC IMPLICATIONS OF US FIGHTER FORCE REDUCTIONS: AIR-TO-AIR COMBAT MODELING USING LANCHESTER EQUATION

I. Introduction

Background

Over the past few decades, the United States military fought comfortably under a blanket of air dominance. In the next few years, the small, almost unnoticeable hole in that blanket grows considerably as its fighter aircraft force decreases, affecting the war-fighters in the air and on the ground and bringing to question its ability to protect itself in support of military policies abroad.

In World War II, the benefits of air dominance took a global stage; enabling the ground forces and naval fleets to enact military might on their foes without regard. In the conflicts since, this theme has been repeated and the importance of controlling the skies has not been overlooked. In the most impressive demonstration of aerial dominance, the coalition air forces of Desert Storm, led by the US Air Force shut down the Iraqi ability to wage aerial warfare, guaranteeing the successful liberation of Kuwait. Victory was delivered by the large US fighter inventory capable of finding enemy aircraft, engaging them beyond visual range (BVR) and employing long range missiles, downing their enemy with unmatched success.

In 1991, the US Air Force fighter inventory numbered 4155 (Ruehrmund and Bowie, 2010:23). This number is significant for two reasons. First and foremost, a large

fighter aircraft inventory allows a military the ability to maneuver aerial forces into key positions and hold them until proven otherwise. Similar to ground schemes of maneuver, aerial maneuver, the ability to intercept enemy aircraft over vast regions relies heavily on appropriately positioning aircraft in anticipation of the enemy's attacks. Secondly, the number of fighters is in direct correlation to the number of missiles available to stop the adversary's forces. Without platforms capable of carrying the BVR weaponry, the ability to secure the skies may be in question. It is troubling to think of defensive forces overrun, or escorting fighters running out of munitions while protecting a strategic bombing campaign.

Today, the US current fighter inventory numbers 2265 and is diminishing due to increasing age and budget constraints (Ruehrmund and Bowie, 2010:25). The lack of fighter numbers is further amplified by the massive reduction in F-22s purchased and increasing delays of the F-35. The newer, stealth fighters are multi-roled, responsible for not only air superiority but precision attack as well, shifting ordnance load outs to bombs in place of air dominance air-to-air missiles. This is significant due to the fact that all weapons are carried internally maintaining the advanced fighters' stealth signatures. For every bomb carried on a combat mission, multiple missiles must be removed from the load out and are unavailable to suppress enemy aircraft.

Concerns do not stop with the shortage of aircraft or missile numbers. The evolution of missile defeating technology and tactics is increasing due to its hugely cost efficient advantages. Electronic attack (EA) targeted at fighter radars and missile seekers is inexpensive, widely proliferated and advancing in complexity at an unprecedented rate.

Unfortunately, the US is still employing the same family of missiles as it did during the Gulf War, albeit carried on lesser numbers of aircraft.

Problem Statement

In the past few decades, the United States government quantified the level of risk it is willing to accept in respect to conflict in the global environment as a function of the numerical strength of the military forces. This level of risk is associated with the ability to combat oppressive forces on two separate fronts, represented by a distinct division of the logistical supply chain and the fighting services. Recently, the risk associated with waging two regional conflicts not co-located increased to a point that is no longer acceptable. The country's leadership decided that the US could no longer support two major wars; instead, the country's military could only fight one large campaign and one smaller, less involved situation. Although quantified by the rough size of the conflict, a specific numerical understanding in respect to the exact force strength required is not established.

Multiple studies have been performed attempting to quantify the state of the US military, specifically air dominance fighters and the strategic implications associated with the decline in numbers. Most research focuses on a specific threat nation or future threat capability, however; none of these studies attempt to employ mathematical methods to empirically solve for the numerical requirements of air-to-air fighters. It is possible to model a benign aerial environment utilizing simulation to isolate certain effects of evolving technology, capture the relevant data and use it as an input to determine the attrition coefficients for a series of Lanchester differential equations. This research

focuses on determining the stochastic effects of electronic attack on a composite force of F-22s and F-15Cs, specifically addressing the number aircraft needed to kill a prescribed number of adversary aircraft. The fighter numbers are modeled to capture expected maintenance availability of aircraft and account for any possible airborne emergencies to determine a specific number of US aircraft required for the air superiority role. These numbers are input into a force-on-force simulation representing three classes of threat numbers and three classes of threat technology levels, representative of their current capabilities. This report attempts to provide mathematical solutions for US fighter requirements for the range of potential conflicts for countries of numerical equality and a high technology state to those at a numerical disadvantage and a low technology state.

Previous Research

This study leverages previous research in two distinct areas of interest, expanding the findings to include mathematical analysis of today's fighter reduction and the strategic implications of a diminished force. In the past, studies have highlighted the dramatic cuts and decreases in fighter numbers and have also noted the considerable decrease in the industrial base and infrastructure required to sustain a military. Ruehrmund and Bowie (2010:5) compares current force numbers to historic levels in Figure 1 and extrapolates conclusions that predict outcomes of the US involvement in potential conflicts for both the short and long term spectrum.

Grant (2009a:3) postulates that strategic consequences are likely to impact the United States' ability to influence the global environment as a result of the forecasted decline in Air Force strength; concluding that the US international policy is threatened as

the Combat Air Force (CAF) finds itself in a growing crisis. "An unstable situation of great danger and difficulty," is the narrative accompanying the critical article describing the growing doom awaiting the United States as its fleet of aircraft reach unprecedented ages, never seen before (see Figure 2).

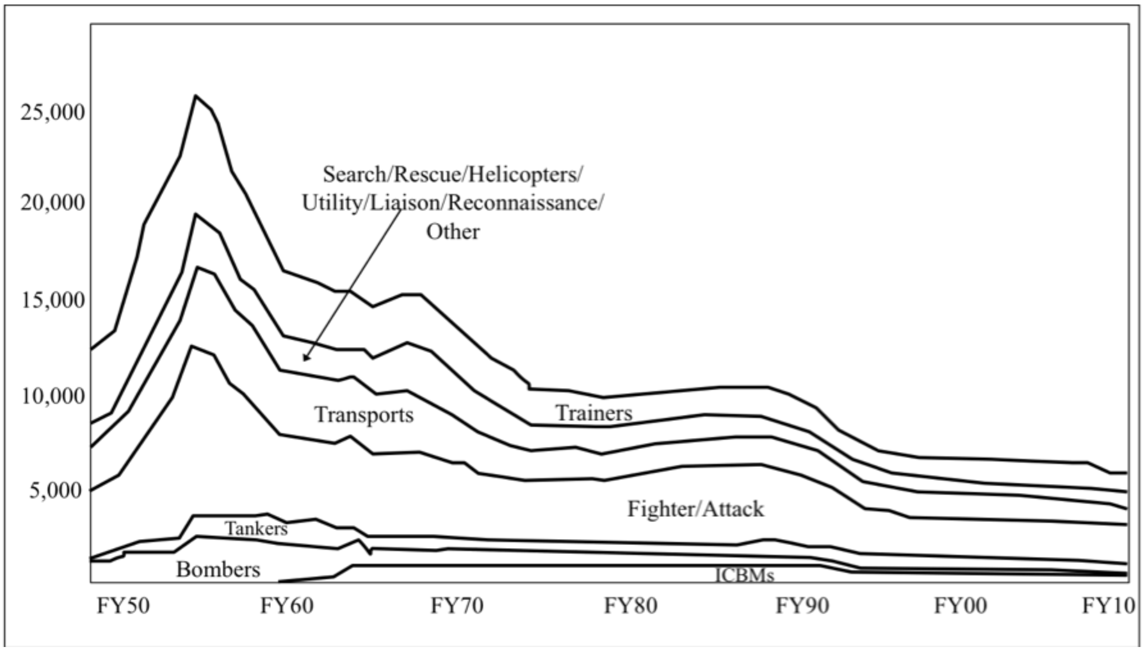

Figure 1. US Aircraft Inventory Levels since 1950 (Ruehrmund and Bowie 2010:5)

Many prominent Air Force leaders are openly speaking about the growing concerns with the US aging fleet and its waning capacity to deter aggression from possible threat countries. Retired Lieutenant General Michael Dunn, current Air Force Association president, recently published the expected cost increases per year of continued operation for the entire aircraft inventory for the White House senior leadership (Dunn, 2011). The research LtGen Dunn presented highlights an additional area of concern besides the obvious reduction in numbers. This increased cost for maintaining the status quo is further hampering the United States' ability to adapt to the

changing global climate and acquire newer, more advanced platforms. As stated by the Under Secretary of Defense for Policy, Michele Flournoy, "the problem of aging equipment is most acute for the Air Force…the service has been conducting combat operations in the Gulf for the past 17 years, patrolling the desert skies. The same 17 years have seen underinvestment in modernization and recapitalization…a financial burden that snowballs with every year," (Dunn, 2011).

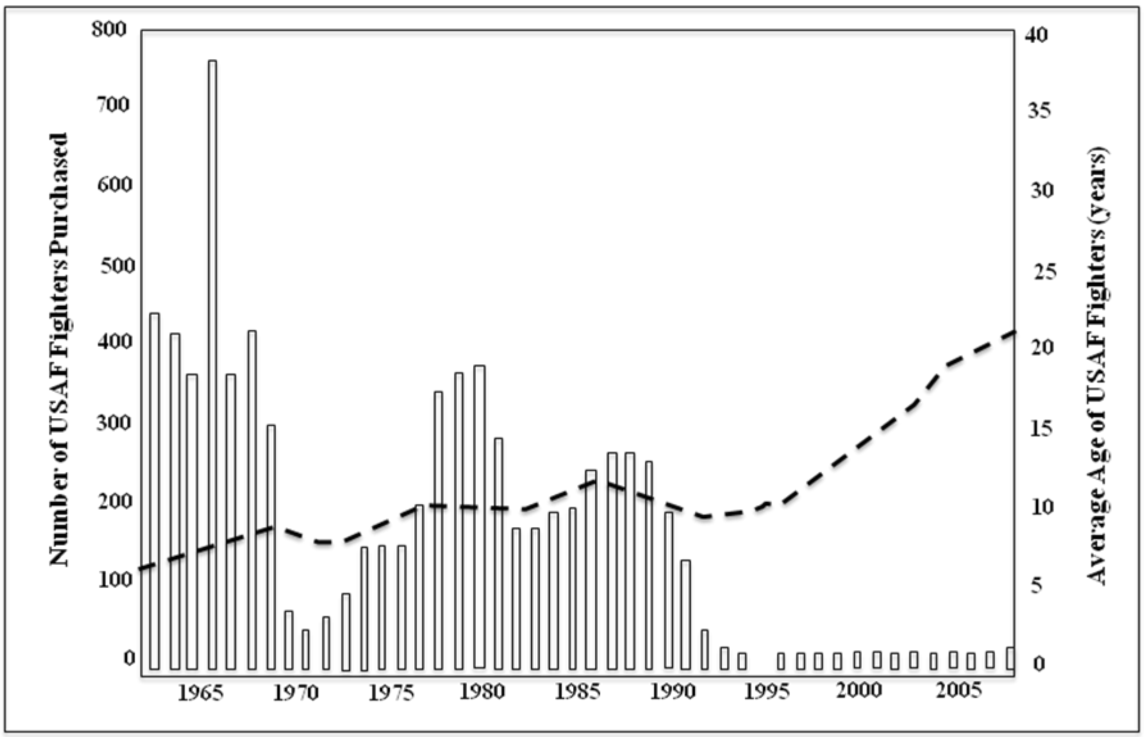

Figure 2. US Fighter Purchases per Year and Average Fighter Age (Grant 2009:21)

The previous Air Combat Command (ACC) commander, Retired General John Corley, stresses the importance of a strong CAF, "USAF global tool sets are necessary to underpin a national military or a national defense strategy, which, in turn, underpins a national security strategy. Global power and global vigilance are where I would start as

we discuss the role of the CAF" (Laird, 2010:1). These comments are echoed by a series of interviews and discussions led by Retired Lieutenant General David Deptula, the former Air Force Chief of Intelligence, Surveillance and Reconnaissance. Most recently, in September of last year, LtGen Deptula met with Defense Secretary Robert Gates and exclaimed, "for the first time, our claim to air supremacy is in jeopardy," further exhorting, "the dominance we've enjoyed in the aerial domain is no longer ours for the taking," (Baron, 2010). His dreadful claims are reinforced by his research into the emerging threat represented by multiple nation states, specifically addressing the leaps in their technology and the US' inability to maintain an equivalent pace (Deptula, 2010).

Table 1. Average Age of USAF Aircraft and Increasing Costs (Dunn, 2011)

Aircraft Type	Average Age (years)	Cost Per Flying Hour Increase FY05-FY09
Fighter	21.1	6.6%
Bomber	34.5	10.5%
Tanker	46.4	5.9%
Strategic Lift	16.5	-0.7%
Tactical Lift	22.9	5.9%
Command and Control	24.6	5.4%
ISR- Manned	23.6	-10.0%
ISR- RPA	3.4	-6.4%
Electronic Warfare	38.0	12.5%
Special Ops	26.6	6.7%
Combat Search and Rescue	25.0	-0.3%
Trainer	23.8	4.9%
Totals	24.0	10.5%

Grant (2009b) argues that greater consequences exist for the extreme nature of the fighter reduction, than those that appear readily on the surface of the discussion. Second and third order effects are already being witnessed as defense organizations downsize

significantly or are forced to merge with other defense companies to maintain solvency. These resultant business effects lead to a drop off in competition for government contracts, which in turn diminishes technological innovation and ultimately decreases a portion of the US' military advantage. Although founded upon historical data, recent trends, and expertise in the field of airpower, none of these studies quantify the scope of the expected decrease in combat capability. Unfortunately, each of the previously undertaken research endeavors lacks a mathematical foundation that elevates their conclusions and recommendations beyond simplistic counting or rhetoric, to definitive, quantifiable, areas of concern. Lastly, the USAF Chief of Staff, General Norton Schwartz, remarked with regard to the change in the US' assumed risk from "low" to "moderate" as a consequence of the aging, decreasing fighter force, "the nature of risk is that airmen will be unable, in a crisis, to successfully carry out their joint missions" (Grant, 2009a:27).

Mathematically, combat analysis has been performed for thousands of years, however; not since the early 1900's have certain methods been understood or available for utilization in military applications. In 1916, Frederick Lanchester developed a simple series of differential equations that explain the relationship of two opposing forces and their abilities to attrit the other over time (Lanchester, 1916: Ch 5). The Lanchester Square Law Equations, listed below assume both forces use aimed fire (threat detected and acquired) and the target acquisition time is independent of the number of targets.

$$\frac{dx(t)}{dt} = -\alpha * y(t) \quad \textit{where } x(0) = X_0\textit{, the initial X-force strength} \tag{1}$$

$$\frac{dy(t)}{dt} = -\beta * x(t) \quad \textit{where } y(0) = Y_0\textit{, the initial Y-force strength} \tag{2}$$

Additionally, α and β, the attrition coefficients, are considered constant over the length of the battle time. Attrition coefficients incorporate all pertinent factors that influence one side's ability to kill the other side (Taylor, 1983).

Lanchester equations are foundational in aggregate force-on-force combat modeling, including specialized equations for varying circumstances and battlefield possibilities. The Square Law equations are manipulated and solved for to include forces utilizing unaimed fire (Linear Law Equations), opposing forces with one side aimed and the other unaimed (Mixed Combat State Equations), forces of varying composition (heterogeneous), and many other forms applicable to specific situations (Taylor, 1983). For the research analysis performed in this study heterogeneous forces are considered, including the ability for the threat Su-27 to detect, acquire, and successfully fire upon a stealthy F-22.

Unfortunately, for complete mathematical appreciation of this problem, it is important to not only grasp the differential Lanchester equations, but to comprehend the attrition coefficients employed that decrement the opposing forces. Multiple methods have been determined to solve for the important coefficients, depending on the known information surrounding each force's capabilities. Taylor (1983) describes a simplistic approach to defining the values, using the known firing rate of a force (υ_y) and the single-shot kill probability of that force on the other warring side (P_{SSK}) as represented below.

$$\alpha = \upsilon_y P_{SSK_{XY}} \tag{3}$$

Although this equation accounts for most simplistic models where each firing outcome is statistically independent and the firing occurs at a uniform rate, it fails to account for important aspects that are common in air combat. Bonder (1967) presents another technique for determining attrition coefficients utilizing a single-shot Markov dependent fire relationship. This equation uses known probabilities of success for each round fired dependent on the success of previous shots. These probabilities are mathematically combined in the equation below to determine the lethality (attrition coefficient) of one force against another.

$$\alpha = \frac{1}{E[T]} = \frac{1}{t_a} + \frac{1}{t_1} - \frac{1}{t_h} + \frac{P(K \mid h)}{t_h + t_f} + \frac{P(h \mid m)}{t_m + t_f} * \frac{P(K \mid h)}{1 - P(h \mid h)} + \frac{1}{P(h \mid h)} - \frac{1}{p_1} \tag{4}$$

Other methods exist for determining attrition coefficients such as the maximum likelihood model that uses a time series of casualties to determine the mean time between casualties and thus overall attrition rate (Clark, 1969).

Regrettably, none of these methods suffice when attempting to apply varying technology and forces' abilities to an attrition coefficient that accurately represents a modern air force. Drew and others takes a different approach towards attrition coefficients. The connection between own force survivability and the opposite forces' attrition is associated. A probability of survival is determined, compared to the number of sorties flown and used as the input to solve for an exchange ratio. This measure of effectiveness describes the number of targets destroyed per own aircraft lost. Drew and others further expands derivations to include factors that increase the probability of survival, diminishing the adversary's ability to attrit own forces. This process for

quantifying attrition coefficients is noteworthy; however, no closed form, easy to replicate or adapt model is presented in their study.

Background Methodology

The approach to modeling the numerical requirements for military forces is not new, however; the application of Lanchester equations to solve for air superiority fighter numbers, explicitly addressing the role of technology is unique. In order to capture relevant results that are of immediate benefit, the scope of the problem is limited. It is anticipated that these results provide a launching point for future research capable of attaining a greater level of depth, particularly related to quantifying other forms of technology as it influences the attrition coefficients. ARENA simulation is employed to model the effects of varying levels of EA on long-range missiles, setting the bounds on numerical expectations in a benign scenario. The resultant data is entered into another model that compares a hypothetical threat force with varied capabilities to determine the probability of victory for the aerial battle. Once success is ensured (within criteria discussed later) the US fighter force strength is varied to highlight the effect air superiority numbers have on expected losses for a potential conflict (see Figure 3).

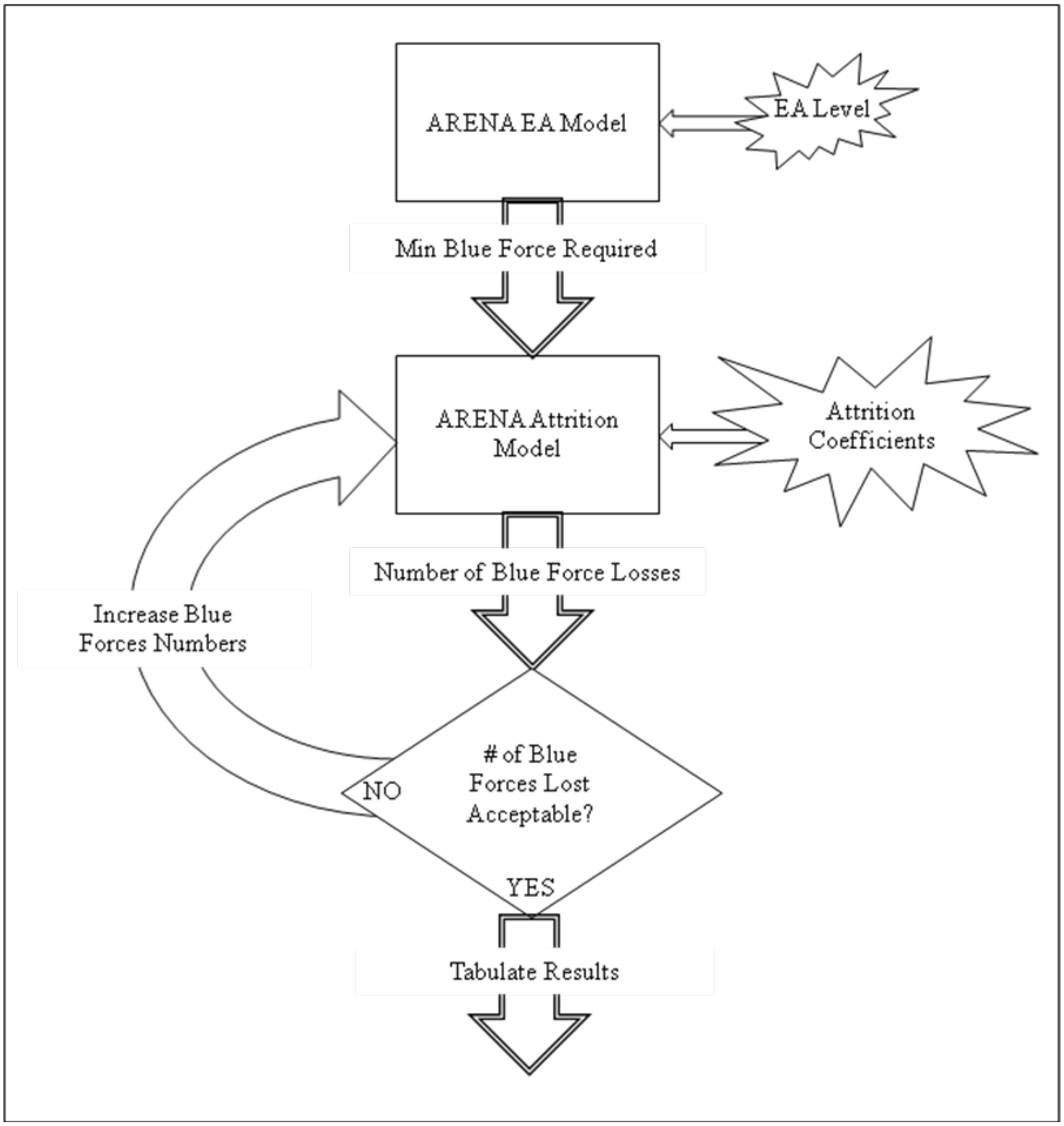

Figure 3. Analysis Methodology

The F-15C exemplifies the counter-air mission that the F-15E, the F-16C/D, and the F-18C-F are all capable of. For the purposes of this study, the F-15C represents all US fourth generation fighters with respect to capability and is loaded out with six BVR missiles and two Within Visual Range (WVR) missiles. The F-22A is considered a fifth

generation fighter with obvious advantages of stealth technology, an advanced sensor suite capable of fusing together multiple sources of battlefield information, and an expanded envelope of flight operation up to 60,000ft and Mach 2.0. The load out for the F-22 varies to characterize the different roles the aircraft is responsible for, however; for this analysis the aircraft is equipped with six BVR and two WVR missiles. These characteristics of fourth and fifth generation fighters are captured in the values for the attrition coefficients.

Table 2. Fighter Aircraft Generation Definitions

Fighter Gen	Era	Capabilities	Examples
1st	Mid 1940s –Mid 1950s	Initial Jet Engines, Machine Gun	Me 262, Meteor, F-80, F-86, MiG-15
2nd	Mid 1950s – Early 1960s	Afterburner, bombs, RADAR, missiles	Mirage III, MiG-21, F-100, F-105, Su-7
3rd	Early 1960s – 1970s	Avionics, LGBs	F-4, F-111, MiG-23
4th	1970s – Present	HUD, HOTAS, FLCS, Pulse-Doppler RADAR	F-15, F-16, F-18, MiG-29, Su-27, Mirage 2000, J-10
5th	2005 - Present	Stealth, Datalink, Sensor Fusion, Advanced Avionics	F-22A, F-35, J-20, Su-PAK FA

The lone threat aircraft modeled in the initial analysis is the Su-27; however, multiple 3rd and 4th generation aircraft are addressed and analyzed later in the case studies. The Su-27 is widely sold to many countries worldwide and is an appropriate representation of a capable foe in the air combat arena. Although its combination of aircraft performance and avionics capabilities places it in the same class as the US fourth generation fighters, the attrition coefficient assigned to the Su-27 is less than the F-15 due to the minimal training and/or incomplete or non-existent tactical development that

international pilots receive on average. The BVR air-to-air missiles utilized by the Su-27 are shorter in range and less reliable than the US capability, further decreasing the lethality of the Su-27 as a platform and impacting the corresponding attrition coefficient.

The scenario modeled begins with an initial number of aircraft from both forces approaching the airspace of contention. There is no background information on a specific threat country or its precise infrastructure of airfields and the geometry between them and the fight. Instead arrival rates are handled as a constant, but may easily be manipulated for application to a defined threat nation or Area of Responsibility (AOR). A significant assumption associated with the entire scenario is that no air refueling is involved on either side, with the exception of representing a forward location that reinforcement fighters may arrive from. This eliminates discussion of aircraft cycling to and from the tanker with varying ordnance and fuel states, dubbed beyond the scope of this investigation. Additionally, both opposing forces are assumed to have access to a base near the area of conflict that facilitates a base of operations and the potential for the arrival of replacement forces as deemed appropriate.

A few important assumptions are made in regard to utilizing the Lanchester differential equations. The mixed composition of F-15s and F-22s representing the United States aerial capabilities typifies a heterogeneous force while the Su-27 threat is simply representative of a homogeneous force. This is an important distinction due to differing mathematical methods employed to handle each case. For this research, Lanchester Square Law models for heterogeneous forces are used and simplified for the Su-27 force. These equations are derived from a few assumptions of their own. The

most critical of these assumptions is that both sides use aimed fire. Although this may seem obvious to most, it is necessary to describe the nature of BVR air-to-air missiles as being RADAR guided, requiring detection and illumination of the targeted aircraft for a valid weapon release. Other Lanchester equation assumptions are lumped into the generality that nothing else is explicitly modeled; instead, the attrition coefficient accounts for all other inputs collectively.

II. Methodology

ARENA Model – EA Effects

The first ARENA model was constructed with the intent to determine the number of blue force aircraft, specifically BVR missiles, required to kill a given number of threat aircraft operating with electronic attack targeted at the F-15 and F-22 composite force. Quantifying the minimum number of missiles needed to attrit an opposing force provides a lower bound for the problem of air superiority fighter requirements. Based on the uncertainty of exact effects that a given EA environment might have, or exact effects resulting from certain techniques, the rate of success for a missile launched against a non-maneuvering, passive target was systematically varied to graphically depict the fighter requirements over the range of potential. This graphical relationship holds true with any software updates to the current BVR missile or the advent of a new missile in the future. The basic scenario created resembles a common scenario flown in US training exercises Red Flag Nellis, Red Flag Alaska, and the Dissimilar Air Combat Training (DACT) portion of the USAF Weapons School syllabi. Twelve F-15Cs and six F-22s are posed against an adversary force of twenty Su-27s. Although numerically representative, the ARENA model lacks the tactical detail and specifics of actual air combat in order to isolate the EA effects on the BVR missiles themselves.

Running the EA Simulation

The aircraft are created at time zero with no replacements planned. Each F-15 and F-22 begins with the standard conventional load-out (SCL) of six BVR missiles. The targets (Su-27s) are assigned to available F-15s and F-22s. Each target is acquired

independently, allocated a missile and subsequently fired upon. For the EA model, no threat maneuvers are simulated, no preference is given between F-15s and F-22s based on current tactics to determine who would shoot, no counter-EA tactics or technology are accounted for, and all missiles are assumed to be semi-active radar (SAR) guided with no modeling of infrared (IR) seeker missiles. Each missile is assigned a probability of success and compared to a random numerical draw to determine if it hits the assigned Su-27. For every missile that hits a target it is assumed to be valid for a kill, eliminating the target from the scenario. If the missile is determined to miss the target, the assigned aircraft shoots another BVR missile and continues to do so until a kill is achieved. There is no change in probability of success, even though theoretically the effects of EA are reduced as the range between the fighter and the threat decrease. This process continues until all 20 threats are destroyed or until all 18 blue force aircraft are out of BVR missiles. This simulation is repeated ten times for each varied level of EA to account for the variance between the random number draws for missile success. The output of the EA model provides the average number of aircraft required for the given blue force (twelve F-15s and six F-22s) to kill twenty threat aircraft for each of the incremented levels of EA modeled. Time, aircraft fuel and aircraft system degradations amongst other possibilities are not captured.

ARENA Model – Lanchester Attrition

The relationship between the EA environment and the number of aircraft needed on average to kill a defined number of threats is input into the Lanchester attrition model as an initial blue force number. Dependent upon the expected EA of a scenario, the

number of initial blue forces is varied to combat the threat numbers. The Lanchester differential equations previously mentioned, equations (1) and (2) are accounted for in the ARENA model, specifically capturing the blue force heterogeneous makeup as witnessed below.

$$\frac{dx(t)}{dt} = -\alpha * y(t) \tag{5}$$

$$\frac{dy(t)}{dt} = -\beta * x(t) - \chi * z(t) \tag{6}$$

$$\frac{dz(t)}{dt} = -\delta * y(t) \tag{7}$$

$x(t)$	=	F-15 force level as a function of time
$y(t)$	=	Su-27 force level as a function of time
$z(t)$	=	F-22 force level as a function of time
α	=	attrition coefficient of Su-27's ability to kill F-15s
β	=	attrition coefficient of F-15's ability to kill Su-27s
χ	=	attrition coefficient of F-22's ability to kill Su-27s
δ	=	attrition coefficient of Su-27's ability to kill F-22s

As highlighted by previous research, the true merit of Lanchester differential equations is the value determined for the attrition coefficient. Accurate attrition coefficients are not only difficult to solve for, they are the most critical element of any force-on-force modeling. Without an accurate representation of the two forces and how they impact the other, the corresponding results have no chance of being useful.

Attrition Coefficient Determination

In an attempt to quantify attrition coefficients applicable to aerial warfare including technological differences amongst the two sides, an initial approach is taken towards identifying a value for the lethality and survivability of each type of aircraft. A

scorecard is created to solve for the value of the attrition coefficient with respect to each type of combatant aircraft. The value is derived from the typical outcome observed at Red Flag Nellis, Red Flag Alaska or the Weapons School DACT phase. These scenarios on average result in 3-4 losses for 4th generation fighters and 1-2 losses for 5th generation F-22s, with a decrease of 20-40% in missile Pk normally encountered. These reoccurring exercise results, coupled with the author's vast experience of over 1500 fighter hours in the F-15C and F-22A and status as an Instructor in the Air Superiority Division of the USAF Weapons School, 433rd Weapons Squadron, are foundational for the determination of the attrition coefficient scoring.

The attrition coefficient is comprised of the advantages or disadvantages in lethality directly compared to the opposing aircraft type and is awarded or deducted points as appropriate. Each aircraft is given extra points, independent of the opposing forces capabilities for technology related to survival: stealth and electronic attack. For aircraft equipped with EA, the amount of points awarded varies with the techniques employed that hinder the opposing forces ability to detect, acquire and/or target with a BVR missile. For every 20% decrease in the opposing forces probability of kill (Pk) due to EA, a greater increment of points is awarded. In addition to lethality and survivability, a time constant (represented by points for both sides) is added to each aircraft to ensure the fight is characterized accordingly with respect to the length of time expected for a typical battle of the given proportions. Lastly, each aircraft receives a constant value for their generational classification (3rd, 4th, 5th) based on their overall design and the era introduced into service; ensuring each aircraft ends up with a coefficient greater than or

equal to zero. In the event an aircraft ends up with an attrition coefficient equal to zero, a nominal value of 0.1 is used instead, to capture the rare possibility that the lesser foe is able to find and kill the greater adversary by sheer chance. The following table outlines the first attempt at providing structure to an otherwise ambiguous, difficult to define measure for force-on-force modeling.

Table 3. Fighter Aircraft Attrition Coefficient Capabilities Scoring

Lethality / Survivability Factors	Scoring for Fighter Characteristics
Fighter Generation	+0.4 per Gen if Advantage / -0.4 if at Disadvantage
TTPs / Training	+0.4 if at Advantage / -0.4 if at Disadvantage
Technology	+0.4 if at Advantage / -0.4 if at Disadvantage
Stealth	+0.5 If Stealth
EA	+0.2 per 20% Degradation of Missile Pk
Time Constant	+0.6 for all aircraft
Fighter Generation Multiplier	+0.2 x Fighter Generation
SUM = Attrition Coefficient	

In the analysis, there are four attrition coefficients used in the differential equations describing the ability of the F-15C to attrit the Su-27, the F-22's ability to attrit the Su-27 and the Su-27's ability to attrit the F-15C and the F-22 independently. The Lanchester equations modeled in this study, do not account for the synergistic effect of the US composite force and is an area for recommended future research due to the significant benefits of stealth and non-stealth fighter integration. Listed below is the scorecard for each of the four separate comparisons and the resulting attrition coefficients.

Table 4. Solving for Attrition Coefficients using Capabilities Scoring

	Su-27 v. F-15C	F-15C v. Su-27	F-22 v. Su-27	Su-27 v. F-22
Fighter Gen	Same = 0.0	Same = 0.0	Advantage = 0.4	Disadvantage = -0.4
TTPs/Training	Disadvantage = -0.4	Advantage = 0.4	Advantage = 0.4	Disadvantage = -0.4
Technology	Disadvantage = -0.4	Advantage = 0.4	Advantage = 0.4	Disadvantage = -0.4
Stealth	No = 0.0	No = 0.0	Yes = 0.5	No = 0.0
EA	20% Decrease = 0.2	20% Decrease = 0.2	None = 0.0	20% Decrease = 0.2
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	4th Generation = 0.8	4th Generation = 0.8	5th Generation = 1.0	4th Generation = 0.8
Attrition Coefficient:	$\alpha = 0.8$	$\beta = 2.4$	$\chi = 3.3$	$\delta = 0.4$

Initial Force Levels

Once the attrition coefficients have been determined, the model assigns initial strength for each of the three aircraft types that are present at the start of the air battle. This number assumes that both sides have plenty of aircraft available to man their initial posture at the desired levels. No maintenance losses are assumed for the initial force strength. Potential maintenance concerns and numeric fall-out are accounted for in the replacement forces. A set number of aircraft is deemed available as reinforcements that are nominally set on a schedule for arrival to the fight, simulating forces that are in theater on the tanker, or at a near base on an alert status. Distance to and from the fight is considered in the arrival time schedule.

Maintenance Modeling

Maintenance troubles are modeled using a random draw for ground maintenance issues combined with a separate random draw that accounts for any airborne emergencies (EPs). These values are determined from the historic data of ground maintenance delivery rates of the F-15 and F-22 as understood from a triangular distribution with a

minimum of 70% aircraft available, a maximum of 100% aircraft available and a mode of 90%. These values are combined with the historic values for airborne EPs uniformly distributed between 1% and 8% to determine if an individual aircraft makes it into the battle.

$$P(Fallout) = \left[(1 - TRIA(0.7, 0.9, 1)) * UNIF(.01, .08)\right]$$

The Su-27 force is decremented in similar fashion; however, due to the well-known maintenance deficiencies abroad, the likelihood of both ground problems and airborne issues are increased slightly, creating a greater chance that an aircraft may not make the fight. Maintenance concerns for fighter aircraft manning is kept constant within the statistical distributions mentioned; however, it is important to note two significant areas not included in the analysis of this report. First, it is possible that aircraft maintenance rates may decrease with time, due to obvious constraints faced in times of conflict and any battle damage that occurs. Second, aircraft lost due to enemy actions are not reflected. This has a significant impact on long-term sustainability of any force and decreases aircraft availability of subsequent missions. Follow-on research is recommended to capture time dependent aircraft availability rates in order to analyze in greater detail the ability to preserve a fighting force over time. The average age of the US' fighter aircraft is a substantial concern as highlighted in the research mentioned earlier. Maintenance modeling is captured in the arrival of reinforcements only not the initial forces, due to the expectation that prepared spare aircraft would be available for the initial fight.

Running the Attrition Simulation

The simulation begins with the initial number of aircraft input directly from the EA ARENA model and steps through the Lanchester equations incrementally as time progresses, constantly determining the current force levels, based on the rate of attrition caused by the opposing force. Attrition continues for both forces until the first reinforcements arrive into the aerial battle. The arrival aircraft immediately supplement the appropriate force, increasing the rate of attrition of the opposite side. The air-to-air fight continues until the termination criterion is met. The simulation ceases when either force reaches zero aircraft remaining. The ARENA attrition simulation is run ten times for each scenario to account for the variance in the maintenance production of aircraft for the reinforcements of both sides. This provides an average number of aircraft lost for the blue forces given a defined EA level and number of threats. The losses are compared to a theoretical value for what is deemed acceptable by US senior leadership based on their policies, as a given percentage of initial force strength. The simulation is repeated again, varying the levels that leadership might accept to lose, solving for the forces required to attain that desired outcome. Finally, the numbers are tabulated for analysis.

In the event, the F-15s and F-22s are destroyed first; the initial numbers are incremented by four F-15s and two F-22s until a winning outcome is achieved. This ratio between the blue fighters is maintained in order to capture the actual tactical formations and employment standards currently trained to and expected to be utilized in the next

conflict. This allows the results to be compiled and compared to the current Air Force F-15 and F-22 squadron strength giving insight into how many squadrons might be required for certain threat countries.

ARENA Model – EA Effects

The first series of ARENA simulation runs define the minimum number of blue force fighter aircraft required to kill twenty non-maneuvering, cooperative targets. The electronic attack emitted by the Su-27s is increased incrementally to gain a mathematical understanding of the impact on the blue fighter force. As witnessed below in Figure 4, EA significantly influences fighter requirements once a certain level of degradation is reached.

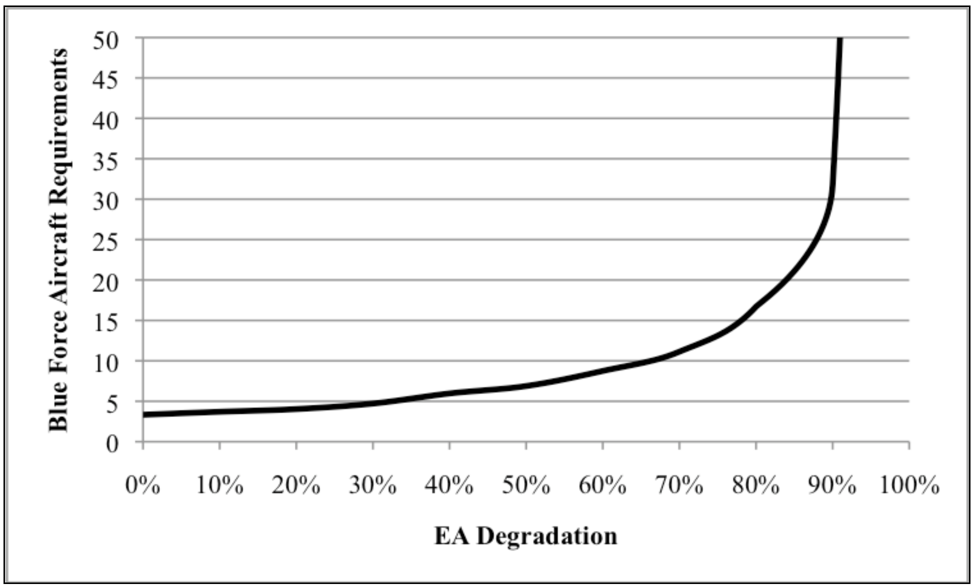

Figure 4. Air Superiority Fighter Requirements to Kill 20 Threats

For the baseline blue force in this study of twelve F-15s and six F-22s, the total numbers are sufficient to handle twenty threats until the missile degradation caused by EA reaches close to 80%. This assumes that the missile separates from the aircraft, guides and functions properly independent of the EA environment 100% of the time, with all failures resultant of threat EA. This assumption is far from reality as most missiles

have some probability of failure associated with many other factors independent of the EA environment. Analyzing fighter requirements directly from the number of missiles needed to kill twenty threats based on the overall missile probability to kill (P_k), accounting for EA and all other effects that might cause a missile to miss its intended target, provides a similar result (see Figure 5).

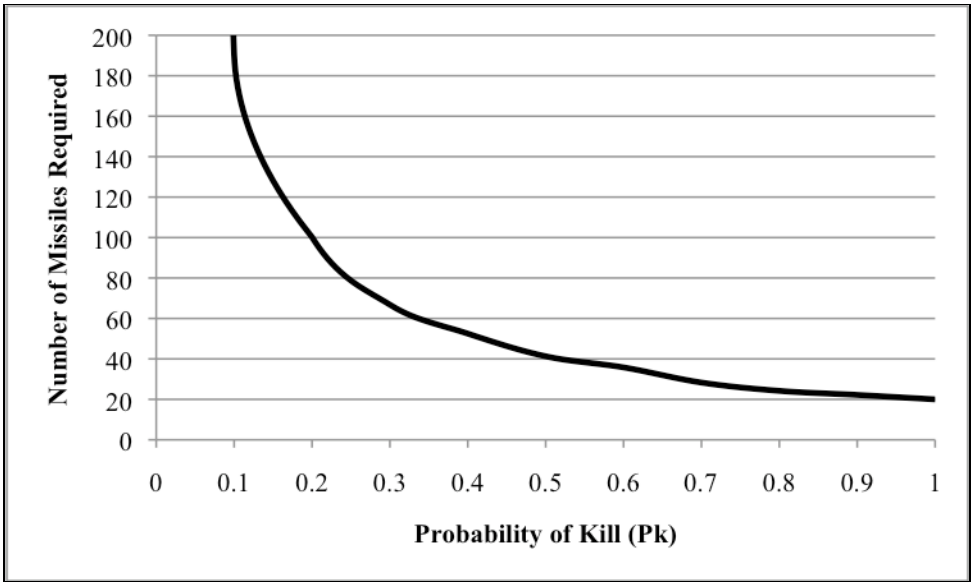

Figure 5. Missile Requirements as a Function of Missile P_k

Figure 5 allows for changing missile technologies, different aircraft types, variances in aircraft weapons load and other variations, assuming only that the same type of missile is analyzed. In order to gain a complete understanding of any specific scenario the known missile P_k independent of EA must be multiplied by the EA degradation expected, solving for the minimum number of missiles required. This number is subsequently used to determine the number of aircraft required dependent on the types and configurations available.

ARENA Model – Lanchester Attrition

Lanchester differential equations are employed to determine the outcome of the two opposing forces, specifically accounting for the technology, tactics, EA and stealth capabilities of each side. The attrition coefficients are solved for using the scoring method previously discussed and iterated for the varying levels of threat EA carried on the Su-27. The ARENA attrition simulation runs begin with the baseline aircraft numbers as output by the EA model (twelve F-15s, six F-22s and twenty Su-27s) with no electronic attack to determine the total blue force losses.

Affects of Increasing EA on Blue Losses

As witnessed in Figure 6, expected blue force losses increase as the EA degradation increases. Of note, there is an appreciable change in the rate of blue losses between 40-50% EA degradation. Additionally, the total losses equal 18 aircraft at 80% EA degradation, representing total blue force annihilation with the baseline initial numbers. Improving the F-15 and F-22 initial force numbers helps lessen the blue force losses as expected. The initial fighter numbers are augmented by four F-15s and two F-22s to decrease the number of total losses by half at an EA degradation level of 80%. This demonstrates the significance of initial force numbers on the total blue losses in an EA environment. Further increases in the initial force strength amplify this result even greater.

The blue losses continue to decrease with each additional four F-15s and two F-22s added to the initial forces. Unfortunately, the decrease is only by two fighters each time an additional six fighters are added. It appears that there is a marginal return for initial numbers of the blue fighter force with respect to the initial threat force numbers.

Tactically, the more assets added, the more difficult it becomes to battle manage the fight and deconflict aircraft; ensuring separation of forces and preventing friendly-fire incidents from occurring. In the planning leading up to a campaign, an assessment should be made on the opposing forces EA capabilities that in turn drive the theater level fighter requirements. Once the EA level of the threat is known, an analysis on force requirements is conducted, comparing the advantages of increasing the force posture and the associated cost with doing so.

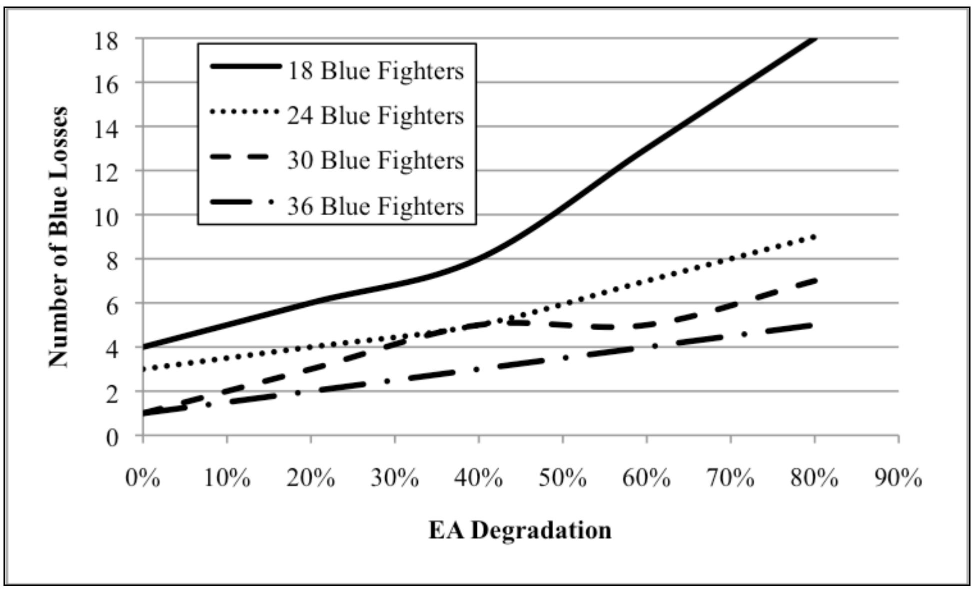

Figure 6. Blue Losses with Increasing EA for Varied Initial Force Strength

Affects of Increasing Initial Fighter Numbers on Blue Losses

In order to capture the exact effect of initial fighter strength, the threat EA is held constant as the initial fighter numbers are increased from the baseline of 18 aircraft to 36 aircraft (24 F-15s and 12 F-22s) representative of two F-15 squadrons and one F-22 squadron on a typical deployment. Figure 7 below highlights the decreased risk of an aerial battle with greater numbers present at the start of the fight.

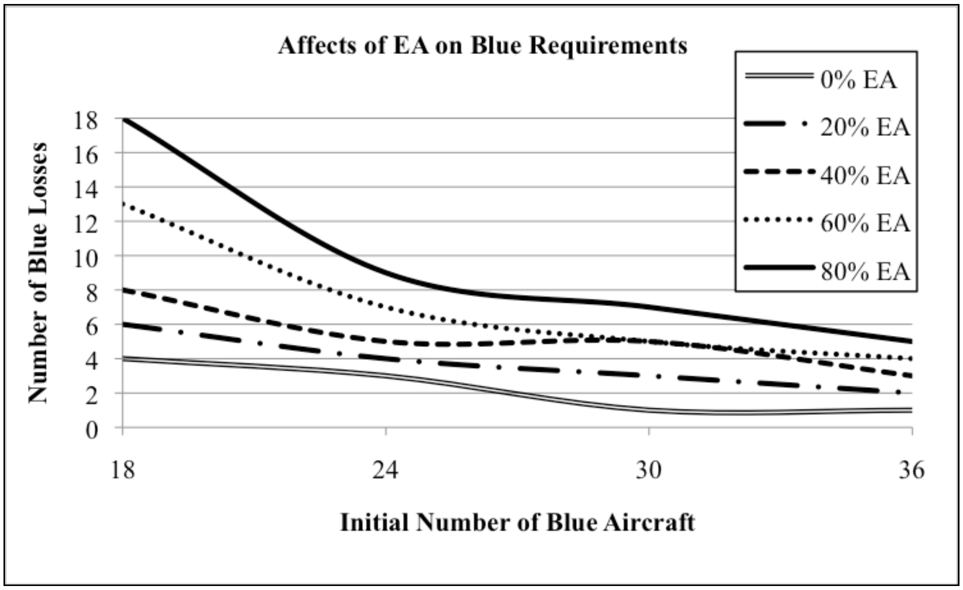

Figure 7. Blue Losses at Varied EA and Increasing Initial Force Strength

Even with no expected EA degradation, the lethality of the opponent affords them the opportunity to shoot down a small number of US forces, assuming they are flying the Su-27 equipped with its standard level of avionics and weapons or an equivalent threat aircraft. The value of increased initial force numbers is apparent as blue losses are minimized to one with 36 initial fighters. Of note, the survivability of the F-22 eliminates all of its losses once the initial force strength reaches 30 fighters (see Appendix B for further graphical representation). It becomes much more difficult to find a stealth aircraft when surrounded by more than twenty non-stealth platforms.

Blue force losses are lessened through greater initial numbers no matter what the expected EA encountered is. The total losses are significantly reduced if the initial blue forces are increased, even if by a small amount. The greater the increase in numbers at the onset of an aerial conflict, the greater the likelihood of blue force success and preservation of assets for follow-on fighting. With 60% EA degradation, the losses are

significant at the baseline force number of 18. An appreciable jump in losses occurs from 40-60% EA, as previously mentioned. This is quantified by 13 total blue force losses, an increase from eight; however, the losses are reduced in half if the initial force numbers are increased to at least 24. This drastic change is characterized by the curve of total blue losses above, specifically the greater slope of the line initially.

At 80% EA total destruction of blue forces is witnessed with the baseline numbers. Once again, an appreciable change in the blue losses is recognized once the initial numbers are increased to 24. This inflection in the curve is important to identify in force analysis and may provide the critical insight that dictates a minimum number of forces required to combat a given adversary with a certain technological state accompanying their numeric potential. This characteristic quantifies a specific ratio of blue initial force fighters to the initial number of threat fighters and is important to capture from simulation with respect to each side's capabilities or more generally their fighter generation comparisons (5th vs. 4th, 4th vs. 4th etc). From the attrition simulations, it is possible to begin drawing conclusions; however, it is important to note that the threat numbers are held constant thus far.

Affects of Increasing Threat Numbers on Blue Losses

Increasing threat numbers with moderate to high levels of EA creates a challenge for any force to counter. Initial numbers must be equal if not greater than the opposing force and other technological advantages must offset the effects presented by EA.
In Figure 8, the total number of threat aircraft is varied until the point of total blue force destruction to determine if there is a critical point for the blue-to-red ratio with respect to

a given EA level. Eighteen total blue fighters (12+6) with no degradation to EA are used as the baseline for comparison. As the threat numbers approach 30, an appreciable increase in blue fighter attrition is noticed. Total destruction of the eighteen fighters occurs at 35 threat aircraft.

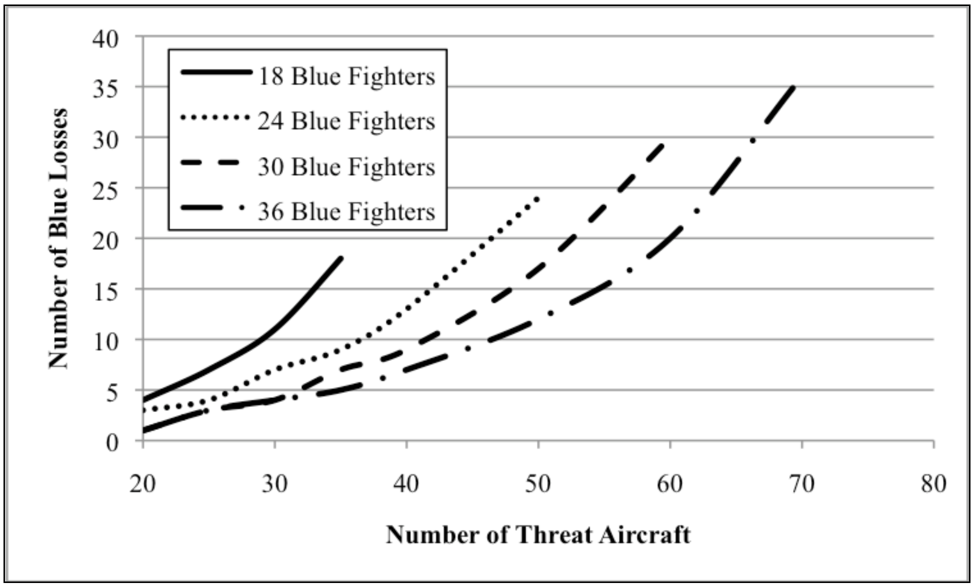

Figure 8. Blue Losses with Increasing Threat Numbers for Varied Initial Fighters

The same simulation is run again, this time increasing the blue force initial posture to 24 fighters. The output data shifts the blue losses significantly as expected, this time with a noticeable increase in blue losses occurring at 35 threat aircraft and total destruction of blue forces at 50 threat aircraft. Each increase in the initial blue fighter strength produces similar results, with an identifiable bend in the curve each time as previously discussed.

Analyzing the same scenario with 30 initial blue fighters (20 F-15s and 10 F-22s) produces a less noticeable increase in blue fighter attrition around 50 threat aircraft. Total destruction takes place at 60 total threat fighters. For 36 initial blue fighters, the

increased attrition point happens around 60 threat fighters with complete blue losses at 70 threats. Two trends in the data are identified when comparing the increasing number of blue fighters and the resultant number of threats capable of being handled. It is apparent that for this scenario, every increment in the initial force of blue fighters by four F-15s and two F-22s delays the characteristic bend in the curve by ten threat aircraft, thus delaying the point of increased attrition. Additionally, it is easily identifiable that total destruction of blue forces occurs at roughly double their initial strength. This graphical relationship may be constructed for any given scenario to capture the critical point in attrition rates and quantifiably assess blue force requirements for any conflict. Although simplistic in nature, it is important to ensure all variables (EA, tactics, training, technology, generational classification of aircraft, as represented by the attrition coefficient) are identified correctly to construct these charts and gain insight on these important correlations.

IV. Case Studies

Scenario Control

With any assessment of a country's military, it is important to concede that numerical representation of their force strength is dynamic and is interpreted as a rough approximation of their capabilities in two varied fashions. First, it is recognized that some countries possess the capability to produce aircraft indigenously and true operational numbers are difficult to capture. Secondly and of much greater significance for analysis of any country, the exact understanding of a military's capability to maintain their aircraft in working order is often difficult to assess. These two areas for potential force disparity should be addressed in any scenario. It is also appropriate to state that all information gained for this analysis comes from open-source publications, void of all classified intelligence channels in order to preserve the distribution of academic material and spawn greater interest in this area of study.

Analyzing a specific country using Lanchester differential equations is difficult to do based on the heavy assumptions required to facilitate the aerial battle. Assumptions on the varied capabilities of the aircraft presented for each of the three countries below must be simplified to produce results compatible with the research presented previously in this analysis. All of the 4th generation fighters per country are considered equal and are added together to produce a single representative number. The same approach is taken for the 3rd generation fighters, lumping them together in order to have two separate numbers representing a simplistic heterogeneous force for each country. The Lanchester equations (5), (6), and (7) derived earlier for the generic scenario must be updated in the

ARENA attrition model to reflect two heterogeneous forces opposing each other, characterized by the appropriate attrition coefficients solved for based on each country's aircraft capabilities (as modeled by fighter generation).

$$\frac{dx(t)}{dt} = -\gamma * z(t) - \eta * w(t) \tag{8}$$

$$\frac{dy(t)}{dt} = -\varepsilon * z(t) - \phi * w(t) \tag{9}$$

$$\frac{dz(t)}{dt} = -\alpha * y(t) - \chi * x(t) \tag{10}$$

$$\frac{dw(t)}{dt} = -\beta * y(t) - \delta * x(t) \tag{11}$$

$x(t)$	=	US 4th Gen force level as a function of time
$y(t)$	=	US 5th Gen force level as a function of time
$z(t)$	=	opposing 4th Gen force level as a function of time
$w(t)$	=	opposing 3rd Gen force level as a function of time
α	=	attrition coefficient of US 5th Gen ability to kill opposing 4th Gen
β	=	attrition coefficient of US 5th Gen ability to kill opposing 3rd Gen
χ	=	attrition coefficient of US 4th Gen ability to kill opposing 4th Gen
δ	=	attrition coefficient of US 4th Gen ability to kill opposing 3rd Gen
ε	=	attrition coefficient of opposing 4th Gen ability to kill US 5th Gen
ϕ	=	attrition coefficient of opposing 3rd Gen ability to kill US 5th Gen
γ	=	attrition coefficient of opposing 4th Gen ability to kill US 4th Gen
η	=	attrition coefficient of opposing 3rd Gen ability to kill US 4th Gen

In order to determine the total number of aircraft available for a potential conflict, a fraction of the total number is used, representative of a force available to the region of conflict based on the vastness of the country analyzed. For the United States' force numbers, a historical representation of fighter units deployed during the Gulf War and subject matter expertise is combined and compared to the current force manning. These numbers are debatable; however, they capture a better understanding towards the true

outcome of force-on-force conflict between two countries, than the total assets. Reinforcements are scheduled similarly, based on the proximity to the perceived battle area and the location to the closest supporting bases or areas for tanker operations. Military planning expertise is used to determine the specifics of aircraft that are dedicated to the initial forces versus reinforcements that are scheduled to arrive from the tanker refueling area. Maintenance modeling is applied as discussed earlier through ten total replications of each case study scenario. The final results tabulated are averages from the multiple replications (see Appendix B for sample data compiled).

The most important take away from this section is not the specifics or the assumptions associated with the given AORs, but the comparison of an adversary with low fighter numbers and minimal technology, a potential foe of moderate numbers and average technology and a country of significant numbers and near-peer capabilities, to the United States. The case studies are not intended for future planning in any of these three areas; instead, they are intended to show application of a mathematical methodology towards an initial understanding on conflict outcome. Military planners with up-to-date intelligence coupled with the assistance of mathematical analysts have the greatest insight into the potential force strengths of the two opposing sides, the reinforcement expectations, the location and regional impacts as well as expected blue force losses. Motivation or strategic reasoning for these conflicts is not discussed and is considered beyond the scope of this analysis.

Venezuela – Low Technology, Low Numbers

The National Armed Forces of the Bolivian Republic of Venezuela are an ideal representation of a smaller country's military taking efforts to establish itself on the global scene as a regional power, attempting to gain respect. Over the past five years, the government took the first steps towards building a credible air defense by purchasing 24 advanced fighter aircraft. In addition to the newly acquired fighters, Venezuela is in ongoing negotiations with Russia to purchase advanced surface-to-air-missile (SAM) systems, further attempting to fortify their defenses. Although Venezuela represents minimal threat to the sovereign United States proper, their anti-American rhetoric is growing, as is their strategic alliance with Iran, bringing to question the future potential for conflict in the area. Listed below are their 3rd and 4th generation aircraft deemed operational at the current time.

Table 5. Venezuelan 3rd and 4th Generation Fighters

Aircraft Type / Generation	Number	Comparable to:
Su-30MKV – 4th Gen	24	F-15E
F-16 – 4th Gen	20	F-16
CF-5 – 3rd Gen	16	F-5
Total:	**60**	

Besides the basic technology that was included on their aircraft platforms, Venezuela does not expand its aircraft lethality or survivability with additional equipment. Most of their military modernization and technological investment, outside of the basic equipped air defenses as mentioned, benefits their Army, including updated personnel carriers, tanks, sniper rifles, night vision goggles and top-of-the-line portable man-carried SAMs. Additionally, their Air Force is limited in tactics and training,

beyond the basic doctrine sold by the Russians. Due to these highlighted deficiencies, Venezuela scores relatively poor when determining their attrition coefficients. As described above, two sets of coefficients are solved for, those describing the rate the United States' 4th and 5th generation fighters (F-15C and F-22A respectively) attrit the Venezuelan fleet and the rate the Venezuelan lesser capable 4th and 3rd generation aircraft attrit the US forces. Listed below are the tables solving for the respective attrition coefficients, utilizing the methods discussed earlier in this paper.

Table 6. Solving for US vs. Venezuela Attrition Coefficients

United States	5th Gen v. 4th Gen	5th Gen v. 3rd Gen	4th Gen v. 4th Gen	4th Gen v. 3rd Gen
Fighter Gen	Advantage = 0.4	Advantage = 0.8	Same = 0.0	Advantage = 0.4
TTPs/Training	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4
Technology	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4
Stealth	Yes = 0.5	Yes = 0.5	No = 0.0	No = 0.0
EA	None = 0.0	None = 0.0	20% Decrease = 0.2	20% Decrease = 0.2
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	5th Generation = 1.0	5th Generation = 1.0	4th Generation = 0.8	4th Generation = 0.8
Attrition Coefficient:	$\alpha = 3.3$	$\beta = 3.7$	$\chi = 2.4$	$\delta = 2.8$

Table 7. Solving for Venezuela vs. US Attrition Coefficients

Venezuela	4th Gen v. 5th Gen	3rd Gen v. 5th Gen	4th Gen v. 4th Gen	3rd Gen v. 4th Gen
Fighter Gen	Disadvantage = -0.4	Disadvantage = -0.4	Same = 0.0	Disadvantage = -0.4
TTPs/Training	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4
Technology	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4
Stealth	No = 0.0	No = 0.0	No = 0.0	No = 0.0
EA	20% Decrease = 0.2	None = 0.0	20% Decrease = 0.2	None = 0.0
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	4th Generation = 0.8	3rd Generation = 0.6	4th Generation = 0.8	3rd Generation = 0.6
Attrition Coefficient:	$\varepsilon = 0.4$	$\phi = 0.0 \Rightarrow 0.1$	$\gamma = 0.8$	$\eta = 0.0 \Rightarrow 0.1$

Conflict Specifics

Venezuela's location along the northern coast of South America lends analysis to hypothesize that a conflict with the United States might occur along the northern border, adjacent to the Caribbean Sea. When analyzing this coastline from an aerial planning perspective it is determined to be roughly 600NM across and represents a significant border to defend with minimal forces. Even with every one of their 3rd and 4th generation fighters operational, their ability to protect their sovereign airspace is nearly zero. This indicates that the capacity to mass forces in a force-on-force scenario is highly unlikely; however, for the sake of this study it is assumed that the Venezuela Air Force fighters are massed to protect their strategic center of operations. Unfortunately, in the time of conflict not all assets are immediately available for a multitude of reasons including long-term maintenance overhaul, non-mission capable due to awaiting parts, aircraft configured for testing, training or other non-mission related duties. It is safe to approximate that Venezuela is challenged to muster 70% of their aircraft listed previously for an immediate conflict. For the simulation, this percentage is applied to their total fighter strength resulting in 31 fourth generation and 11 third generation fighters (totaling 42) plugged into the ARENA attrition simulation for each of the ten replications.

The United States fighter inventory more than suffices for a future conflict of this type and is able to handle the numbers without concern. Staging operations would occur in a nearby allied country, with enough aerial refueling tankers to handle the small requirements. At least 24 F-15s and 12 F-22s would be used for the initial fight, with

reserves airborne on the tanker. These numbers are used as the US forces in the ARENA attrition simulation opposing the Venezuelan numbers mentioned above.

Venezuela – Analysis and Results

Table 8 below outlines the averages from the ARENA attrition model. These results represent an initial planning estimate towards any conflict the two opposing forces may engage in. There are many elements of warfare that are not accounted for; however, these results provide an initial relationship between the Venezuelan Air Force and the US Air Force fighter capabilities. As highlighted in Section III of this report, increasing US fighter numbers initially further decreases the potential for blue losses in this conflict.

Table 8. US vs. Venezuela ARENA Attrition Simulation Results

Initial Blue Forces	Reinforcements	F15 Losses	F22 Losses	Total Blue Losses	Threats Killed
36	5	1	1	2	41**
** Simulation terminated for Total Red Force Destruction					

The US initial forces of 24 F-15s (or 4th generation equivalent) and 12 F-22s easily handle the Venezuelan numbers as expected. The reinforcement schedule for the simulation accounts for possible maintenance troubles and probable distances from the aerial battlefront, allowing for five US fighters (four F-15s and two F-22s) to enter the fight as well as 17 (out of a possible 18 total) Venezuelan fighters launched from potential strip alert positions. Unfortunately, the simulation totals two blue losses for the US, due to the initial number of Venezuelan 4th generation aircraft. Although the possibility exists for blue losses in this AOR and the importance of preparation for such a scenario should not be overlooked, other complimentary military assets might prevent any losses from actually occurring. The US maintains the advantage of surprise for any

conflict that takes place on foreign soil or above foreign territory. This conflict would more than likely consist of cruise missiles or other standoff weapons, alongside of space and cyberspace assets targeted at strategic nodes intended to weaken the Venezuelan political and military structure; occurring simultaneously with the first US aircraft being launched. This surprise greatly impacts Venezuela's fighter force's ability to find and target US aircraft before being completely destroyed. The blue loss numbers that are output from the ARENA attrition model should be treated as simply the result of force-on-force conflict, absent of the other noteworthy advantages the US military maintains over its adversaries and represents more closely a worst-case outcome.

IRAN – Moderate Technology, Moderate Numbers

The Islamic Republic of Iran Air Force represents a handful of countries worldwide that have a moderately sized force utilizing some elements of technology that enhance their lethality and/or survivability. Their government is aligned in opposition to the United States and takes opportunity to voice their dissent on a regular basis. There is a growing concern from the global populace that Iran is taking measures towards securing nuclear capabilities, specifically efforts directed towards the creation of atomic weapons. This represents a huge source of instability for the already volatile region and increases the likelihood for future US involvement.

The Iranian Air Force is a hodge-podge of aircraft and capabilities resultant of multiple conflicts and previous military acquisitions. Its composition, including previously exported US fighters, is somewhat unique in regards to the potential threat facing the US. As initially stated in the scenario control section, it is most difficult to

assess the operational status of these antiquated US exports due to the termination of all parts and maintenance to Iran from the US many years ago. Another significant portion of their fleet is remnant of the Gulf War in 1991, with multiple Iraqi aircraft acquired as defectors during the conflict. None of these aircraft included any maintenance or replacement parts and are questionable in their operation. Iran is in constant contact with the Russian military industry and is suspected of attempting to acquire modern fighters; however, as of this publication, none are known to have exchanged hands. The most updated list of their 3rd and 4th generation fighters is given below.

Table 9. Iranian 3rd and 4th Generation Fighters

Aircraft Type / Generation	Number	Comparable to:
MiG-29 – 4th Gen	40	F-15C
F-14 – 3rd Gen	25	F-14
F-4 – 3rd Gen	65	F-4
Mirage F-1 – 3rd Gen	24	F-4
Total:	**154**	

Based on their continued relationship with Russia and their ongoing arms discussions, Iran is accumulating a more robust air defense, specifically surrounding their populated industrial regions. The technology is believed to be moderate in nature and may even include some indigenously built and designed systems. In addition to Russia and Belarus, Iran trades technology with China, North Korea and even Brazil, potentially bolstering their older air force with newer equipment. Unfortunately, not much is publicized openly about most of their capabilities and it is difficult believing what is released; however, it is fairly safe to say that they have not received any new fighter aircraft and have not produced any internal to Iran. Listed below are the two sets of

attrition coefficient scorecards. Of note, the Iranian Air Force receives slightly higher values than Venezuela due to suspected EA on some of their 3rd generation fighters.

Table 10. Solving for US vs. Iran Attrition Coefficients

United States	5^{th} Gen v. 4^{th} Gen	5^{th} Gen v. 3^{rd} Gen	4^{th} Gen v. 4^{th} Gen	4^{th} Gen v. 3^{rd} Gen
Fighter Gen	Advantage = 0.4	Advantage = 0.8	Same = 0.0	Advantage = 0.4
TTPs/Training	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4
Technology	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4
Stealth	Yes = 0.5	Yes = 0.5	No = 0.0	No = 0.0
EA	None = 0.0	None = 0.0	20% Decrease = 0.2	20% Decrease = 0.2
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	5^{th} Generation = 1.0	5^{th} Generation = 1.0	4^{th} Generation = 0.8	4^{th} Generation = 0.8
Attrition Coefficient:	α = 3.3	β = 3.7	χ = 2.4	δ =2.8

Table 11. Solving for Iran vs. US Attrition Coefficients

Iran	4^{th} Gen v. 5^{th} Gen	3^{rd} Gen v. 5^{th} Gen	4^{th} Gen v. 4^{th} Gen	3^{rd} Gen v. 4^{th} Gen
Fighter Gen	Disadvantage = -0.4	Disadvantage = -0.4	Same = 0.0	Disadvantage = -0.4
TTPs/Training	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4
Technology	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4
Stealth	No = 0.0	No = 0.0	No = 0.0	No = 0.0
EA	20% Decrease = 0.2	20% Decrease = 0.2	20% Decrease = 0.2	20% Decrease = 0.2
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	4^{th} Generation = 0.8	3^{rd} Generation = 0.6	4^{th} Generation = 0.8	3^{rd} Generation = 0.6
Attrition Coefficient:	ε = 0.4	ϕ = 0.2	γ = 0.8	η =0.2

Conflict Specifics

The geographic layout of Iran is much greater than that previously discussed with Venezuela. Any conflict held in the Iranian airspace would initially be localized to achieve air superiority only during the window of ground strikes by conventional (non-stealth) bombers. Air superiority forces would withdraw until the next series of strikes and then establish local air superiority again and would continue to do so, until the Iranian air defenses were softened, strategic targets were destroyed and forward basing of fighters could take place, allowing for continuous air superiority. This approach to gaining air dominance is the most likely option for AORs of massive scale. Although the

Iranian territory is spread out, it is safe to conclude that most of their fighters are centralized around their strategic centers of gravity, specifically around Tehran and Esfahan. A potential aerial battle might take place on the outer perimeters of this central region, consisting of a large number of the Iranian forces that are operational. Seventy percent of the fighters listed in Table 9 are assumed to be available, divided into initial forces and reinforcements from the surrounding bases. This force strength of 108 fighters is easily contested; however, the salient points of discussion involve an increase in numbers and some increase in technology over the previous example.

Contrary to the Venezuelan case study, the United States total numbers involved would be much greater; however, those initially dedicated to any specific mission such as gaining air superiority over a localized area, would be of similar nature, 24 F-15s and 12 F-22s. These numbers are used for the initial US force strength; however, a significant increase in reinforcement numbers is simulated representing greater forces available for the larger conflict. These forces would come from a nearby aircraft carrier, if close to one of the seas, or direct from a nearby air-refueling track.

Iran – Analysis and Results

Differing from Venezuelan forces in a few facets, Iranian fighters represent a larger numeric force, with greater potential for some technology influence. Based on the expected AOR scenario, the same number of blue forces is employed, however; greater numbers are available as reinforcements. Table 12 below lists the expected outcome (averages) of an aerial battle with Iran within the prescribed constraints.

Table 12. US vs. Iran ARENA Attrition Simulation Results

Initial Blue Forces	Reinforcements	F15 Losses	F22 Losses	Total Blue Losses	Threats Killed
36	6	3	2	5	64**
** Simulation terminated for Total Red Force Destruction					

It is apparent from the Section III analysis that greater threat numbers result in a larger number of blue total force losses. Larger threat numbers are coupled with the expectation for slightly more EA, specifically equipped on the 3rd generation aircraft, unlike Venezuela. These inputs increase the blue losses over the previous case study to five. The total number of threats killed is 64 resulting in the localized fighting to be terminated and air superiority to be established by the US fighters. Due to the time elapsed, six US reinforcement fighters (four F-15s and two F-22s) entered the fight, while 14 threat fighters scrambled from their ground alert to join the battle and subsequently be shot down. As discussed with the Venezuelan analysis, an increase in the initial numbers of US fighters directly decreases the expected blue losses. Additionally, it is important to once again consider that any conflict over Iran is initiated on the terms of the US, employing its full arsenal of military capabilities that significantly impacts the Iranian force's ability to launch and direct their fighters appropriately.

China – High Technology, High Numbers

The Peoples Republic of China represents the worst possible scenario for the United States military in many circumstances. The rapid evolution of their military is unmatched for nearly a decade now, laying the foundation for advanced capabilities in the air, on the land, and patrolling the seas. In addition to their conventional military forces, their expansion into the domains of space and cyberspace bolsters their global

presence and cements their dominance in the Pacific. Their current strength is arguably on par with the United States' military's ability to wage war in the region; however, the rate of expansion into developing technologies is without equal and may soon (if not already) fortify their position at the top in the global arena. China's level of technology is considered amongst the highest, placing them in the top tier of world countries and their aircraft numbers, comprised of the People's Liberation Army Air Force (PLAAF) and People's Liberation Army Navy (PLAN) assets exceed all but the United States currently.

Table 13. Chinese 3rd and 4th Generation Fighters

Aircraft Type / Generation	Number	Comparable to:
Su-30MKK/MK2 – 4th Gen	127	F-15E
Su-27SK/J-11B – 4th Gen	132	F-15C/F-15E
J-10 – 4th Gen	80	F-16
J-8 – 3rd Gen	390	F-4 / MiG-23
J-7 – 3rd Gen	579	F-5 / MiG-21
JH-7 – 3rd Gen	70	F-111
Total:	**1378**	

Listed above is a summary of China's fighter aircraft, not including any bomber, reconnaissance, light attack, command and control, or other aircraft of various utilities. Only 3rd and 4th generation aircraft are listed; however, it is important to note, that China is currently in development of their first 5th generation fighter, the J-20, which is claimed to be similar in capability to the F-22A, employing stealth technology, advanced integrated avionics and a sensor suite that allows for increased battlefield awareness. Additionally, it is rumored that China already has or is currently in the process of converting hundreds of their 1st and 2nd generation fighters into unmanned drones, intent

on saturating their skies simply to absorb an opponent's limited number of air-to-air missiles prior to any force-on-force conflict.

Determination of the appropriate attrition coefficients for China requires an additional increase over Iran in their ability to generate degrading EA. Although it is assumed for this example that their degradation tops out at 40%, it is highly possible that it extends well beyond this measure and may have much more significant impact than that credited in the scoring below. Additionally, the extent of their tactics and training is not widely known; yet, it is admittedly greater than either two of the previous case studies and is potentially increasing as well. Other synergistic effects represented by a complete force encompassing, air, land, sea, space and cyber domains are omitted and represent a significant potential, although unquantifiable, for both sides in this specific scenario.

Table 14. Solving for US vs. China Attrition Coefficients

United States	5^{th} Gen v. 4^{th} Gen	5^{th} Gen v. 3^{rd} Gen	4^{th} Gen v. 4^{th} Gen	4^{th} Gen v. 3^{rd} Gen
Fighter Gen	Advantage = 0.4	Advantage = 0.8	Same = 0.0	Advantage = 0.4
TTPs/Training	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4	Advantage = 0.4
Technology	Advantage = 0.4	Advantage = 0.4	Same = 0.0	Advantage = 0.4
Stealth	Yes = 0.5	Yes = 0.5	No = 0.0	No = 0.0
EA	None = 0.0	None = 0.0	20% Decrease = 0.2	20% Decrease = 0.2
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	5^{th} Generation = 1.0	5^{th} Generation = 1.0	4^{th} Generation = 0.8	4^{th} Generation = 0.8
Attrition Coefficient:	$\alpha = 3.3$	$\beta = 3.7$	$\chi = 2.0$	$\delta = 2.8$

Table 15. Solving for China vs. US Attrition Coefficients

China	4^{th} Gen v. 5^{th} Gen	3^{rd} Gen v. 5^{th} Gen	4^{th} Gen v. 4^{th} Gen	3^{rd} Gen v. 4^{th} Gen
Fighter Gen	Disadvantage = -0.4	Disadvantage = -0.4	Same = 0.0	Disadvantage = -0.4
TTPs/Training	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4	Disadvantage = -0.4
Technology	Disadvantage = -0.4	Disadvantage = -0.4	Same = 0.0	Disadvantage = -0.4
Stealth	No = 0.0	No = 0.0	No = 0.0	No = 0.0
EA	40% Decrease = 0.4	40% Decrease = 0.4	40% Decrease = 0.4	40% Decrease = 0.4
Time Constant	= 0.6	= 0.6	= 0.6	= 0.6
Fighter Gen Multiplier	4^{th} Generation = 0.8	3^{rd} Generation = 0.6	4^{th} Generation = 0.8	3^{rd} Generation = 0.6
Attrition Coefficient:	$\varepsilon = 0.6$	$\phi = 0.4$	$\gamma = 1.4$	$\eta = 0.4$

Conflict Specifics

If military conflict were to occur with China, it is highly probable that it would occur in the straits of Taiwan, off the eastern coastline of the mainland. Although this stretch of territory is significant, a potential for a force-on-force clash might occur over the narrow body of water directly between the two coasts. The considerable numbers of 4^{th} generation fighters that China currently maintains would be heavily represented and available utilizing forward deployed bases strategically located at varying locations along the eastern coast. These large numbers alongside of the even greater representation of 3^{rd} generation fighters would be controlled operationally to scale the fight as deemed appropriate for their military objectives. Of note, the largest limitation for this specific AOR is the distances associated from the locations of prospective basing for US forces. The ability of the US forces to maintain any posture over any time period in such a conflict is not analyzed, but must be addressed in any military planning that is performed.

The lower bound on this analysis addresses 100 4^{th} generation aircraft complemented with 100 3^{rd} generation aircraft plus 50 of each in reinforcement status for China. On the upper bound, the numbers may be increased to represent a much greater

portion of the Chinese force, although as determined below, this is largely unnecessary due to consistent conclusions reached at the smaller force level. The US numbers represent eight F-15C squadrons (4^{th} generation) and four F-22 squadrons (fifth generation) deployed to locations as available near the AOR. This number of squadrons is able to support 48 F-15s and 24 F-22s for the initial fight and half of that (24 F-15s and 12 F-22s) in reserve on a nearby tanker.

China – Analysis and Results

A significant disparity is noticed immediately with the output of the ARENA attrition model for China in comparison to the other two case studies. Based on the sizeable initial force of 3^{rd} and 4^{th} generation fighters and the increased technological capability that presents itself in greater EA degradation, China is able to defeat the entire blue force of 64 F-15Cs and 32 F-22s (initial force and reinforcements). This result leads to running the simulation again with increased initial blue forces and results in similar output. The second set of data highlights total destruction of the US fighters once more; however, the increased blue numbers delay the outcome allowing more reinforcements to arrive, while destroying almost twice as many Chinese aircraft. This iteration of blue initial strength is repeated to determine the point of inflection, representing the smaller blue force attrition rate as demonstrated in Section III. Captured below in Table 16 are the average results of each of the five separate tests, run ten times each to account for the stochastic effect of maintenance variation, each representing a different blue force initial strength.

Table 16. US vs. China ARENA Attrition Simulation Results

Initial Blue Forces	Reinforcements	F15 Losses	F22 Losses	Total Blue Losses	Threats Killed
72	24	64	32	96*	103
90	36	84	42	126*	221
102	36	56	27	83	248**
108	36	48	24	72	248**
126	24	35	17	52	248**

* Simulation terminated for Total Blue Force Destruction
** Simulation terminated for Total Red Force Destruction

Although the total blue forces lost in each of the simulation runs is still considerably larger than the previous two case studies, a definitive point of decreased blue attrition is witnessed. The number of blue losses compared to threat losses changes significantly from the first simulation of 72 initial blue fighters to that of 126 initial blue fighters for the last run. A ratio of nearly 1:1 for blue to red losses decreases to almost 1:5 with the increase in initial numbers. The threat losses increase two and a half times to a total of 248 fighters from 103, over the sample space of data collected. Further threat forces entered into the fight (presumably from alert or off a threat tanker) would only increase their losses. Once initial fighter strength attrits the main body of the threat force, the ability for the threat force to recover is nearly impossible with the expected rate of any reinforcement force. Figure 22 highlights the point of decreased attrition rate as the initial blue fighter strength approaches 100 fighters, with respect to the 200 initial Chinese fighters.

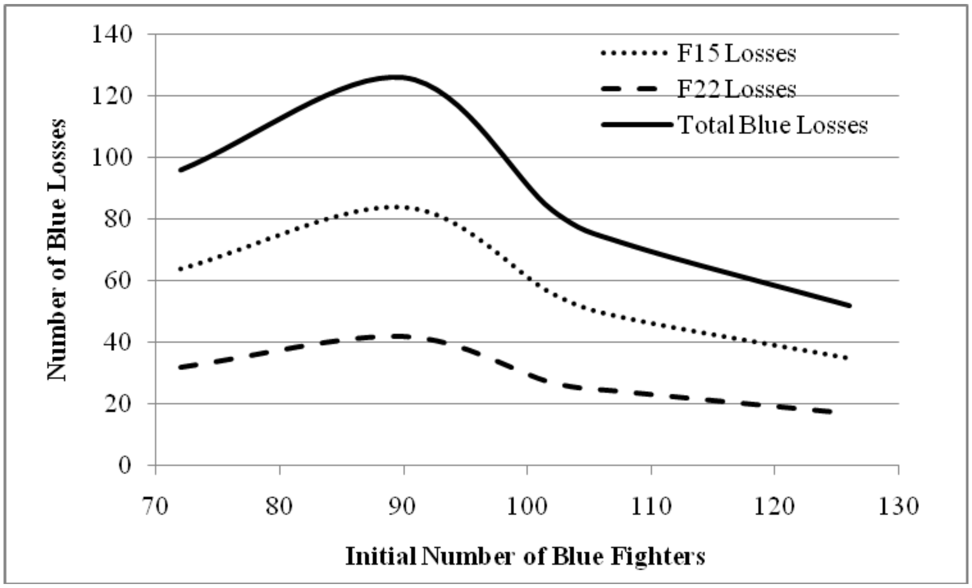

Figure 9. US Blue Losses vs. 100 3rd Gen and 100 4th Gen Chinese Fighters

This approximate ratio of one initial blue fighter for every two initial Chinese fighters (one 4th generation and one 3rd generation) represents the merit of the ARENA attrition model and is recommended to be used to extrapolate the US fighter requirements for conflict with China. If it is expected that the Chinese initial force consists of 200 initial fighters, than at least 100 US fighters should encompass the initial force to achieve the lesser attrition rate. It is important to note; however, that although a lesser attrition rate is realized, significant losses are expected in any conflict with the Chinese unless the numerical advantage shifted towards the US direction. Unfortunately, this possibility is severely limited by the US' ability to deploy the massive numbers needed to offset the Chinese potential. If China forward positioned half of their 3rd and 4th generation fighters, the US would need nearly 700 fighters in the AOR to equal the opposing strength. This represents a near impossibility due to many logistical challenges that have no solutions.

V. Conclusions and Recommendations

Mathematical analysis provides a foundation for the political and military decisions surrounding fighter force manning, specifically the number of aircraft required to combat given threat scenarios. As demonstrated in the case studies of Venezuela and Iran, the current force structure is adequate and continues to be sufficient for any conflict representative in nature of low to moderate fighters numbers with low to moderate technology and capabilities. Although the US' force composition will continue to decrease over the next five years as the F-15, F-16 and F-18 fleets diminish, taking the fighter inventory to historical lows, the US' ability to project presence into AORs of low to moderate posture remains uninterrupted. Once the eventual F-35 production is at full strength and substantial numbers are rolling off the production line annually, the US' fighter strength will be renewed to match that of the past thirty years.

Unfortunately, none of this optimistic projection answers the present numeric challenges that are at hand with China or any other country that demonstrates technological advances alongside of growing inventory. Although the list of countries equipped with high numbers and high technology is fairly limited, arguably Russia, India and the aforementioned China, the evolution of tactical and operational capabilities may marginalize the United States' fighter force's strength altogether. Certain environments are growing less advantageous and the US' global might is slipping as it fails to account for the expanding threat basis. Mathematically, there are various assumptions that accompany any analysis; however, even if the assumptions represented in the attrition coefficient determination are tilted towards the US favor and if the initial fighter

numbers, reinforcement numbers or reinforcement arrival schedule are adjusted to the benefit of the US, the outcome is still highly undesirable and the consequences are undeniably grave. This quantifiable understanding provides structure to the previous rhetoric of declining fighter numbers and the resultant declining strategic global influence. There is no refuting the military implications of a lesser force in this circumstance.

All is not lost in respect to the growing hegemony represented by China, India or that previously understood by Russia. Although the US acquisition timeline is disadvantageous with respect to impacting the sliding numerical difference between fighter aircraft, technology may be utilized to maintain its battlefield advantage. Advancement in missile technology to ensure survivability in a dense EA environment increases the effectiveness and produces more air-to-air victories per missile fired. Limitations to the currently fielded BVR missiles are discussed at length in open-forum. Advances in guidance logic makes it possible to target an identified threat with off-board sensors that are dispersed, vast and inter-connected such that no missile guidance is reliant upon a single sensor that may be easily jammed, causing failure. This off-board cueing takes advantages of multi-spectral sensors comprising a network of platforms that collectively build, maintain and map out an entire battlefield, in the air and on the ground. Most of this technology is inexpensive comparatively and represents a redundant system of varied capabilities overlapped to ensure constant coverage of the desired area, providing consummate battlefield situational awareness.

Additional areas for technology expansion include but are not limited to: increased missile kinematics, smaller missiles of similar capability, increased electronic

attack whether internally configured or externally mounted, multi-spectral sensor utilization such as an IRST on most fighters, and unmanned platforms of varying capacity. Spending drastically less money and acquiring advanced capabilities in a considerably shortened period is an answer to the current economic constraints and the impending decrease in the US' numerical representation. These technology developments directly impact the lethality or survivability of US forces, providing a new tactical margin on the battlefield and hopefully accounting for any disparity encountered.

Further mathematical study is recommended, employing various techniques to better quantify the growing problem and addressing potential solutions to the United States' air superiority concerns. Application of Lanchester differential equations to anticipate potential blue force attrition for actual CONPLANs and OPLANs, would refine the next level of analysis and better describe the actual requirements. Attrition coefficient determination using scoring methods as discussed in this research could be built upon drawing greater detail from existing capabilities, potentially applying a weighting scale to those characteristics more crucial to survival or those that significantly increase lethality. This weighting of characteristics might better represent the actual threat environment operating in, accounting directly for adversarial capabilities in true sense instead of simplistic 3rd and 4th generation generalities. The Lanchester differential equations could be supplemented to capture threat surface-to-air missile numbers, location and capabilities, adding complexity to this basic research that would better quantify the results from one AOR to another. Lastly, all assumptions for force composition, capabilities, reinforcement schedules and basing expectations could be

addressed, albeit, on the classified level further enhancing the understanding between the US' capabilities and that of a given threat nation.

Not since Korea, has the United States been forced to contend with parity in the realm of air superiority; there is no reason it should settle for equality now. The US military forces have operated under a blanket of uninterrupted protection for decades, leveraging its innovation and creative mind in the face of adversity. It is important that the US continues to adapt to the global scene and mitigates the impending gap in its fighter posture. As a nation the US must employ the critical understanding gained through mathematical analysis as its foundation for objective discussion vice empty, politically fueled arguments that amount to nothing more than rhetoric, representing unbounded possibilities that stray aimlessly from the truth, adding to the problems instead of taking the necessary steps towards solving them.

In conclusion, the US is capable of fighting two (maybe more) small conflicts with country's that are numerically inferior and technologically handicapped; however, any argument surrounding the ability to fight more than one force of moderate strength and moderate technology should be heavily reconsidered. Most troubling is the realization that the US is unable to fight an adversary of equal or greater strength unless it is willing to invest its entire military system towards the logistical feat. Even then, mathematical analysis recommends against this conflict due to the significant losses expected. This analysis quantifies the previously ambiguous risk associated with US military conflicts and its ability to stretch its forces across multiple AORs.

Appendix A. List of Abbreviations and Acronyms

ACC – Air Combat Command

AOR – Area of Responsibility

BVR – Beyond Visual Range

CAF – Combat Air Forces

CONPLAN – Conceptual Plan

DACT – Dissimilar Air Combat Training

EA – Electronic Attack

EP – Emergency Procedure

F – Fighter

FLCS – Flight Control System

HOTAS – Hands on Throttles and Stick

HUD – Heads Up Display

IR – Infrared

IRST – Infrared Search and Track

LGB – Laser Guided Bomb

LtGen – Lieutenant General

Me – Messerschmitt

MiG – Mikoyan Gurevich

OPLAN – Operation Plan

P_k – Probability of Kill

RADAR – Radio Detection and Ranging

SAM – Surface-to-Air Missile

SAR – Synthetic Aperture RADAR

SCL – Standard Conventional Load

Su – Sukhoi

TRIA – Triangular Distribution

UNIF – Uniform Distribution

US – United States

USAF – United States Air Force

WVR – Within Visual Range

Appendix B. Amplifying Data from ARENA Simulation Results

Table 17. Sample Data Collected from EA Model

Simulation Run #	Model	Testing	EA Level	Threat Numbers	Test Coefficients	Total Blue Numbers	F15 Losses	F22 Losses	Total Blue Losses	Time (hours)
1	EA Model	Increasing EA	0%	20	a = 0.6, b = 2.4, c = 3.3, d =0.2	12 + 6	3	1	4	0.5
2	EA Model	Increasing EA	0%	20	a = 0.6, b = 2.4, c = 3.3, d =0.2	16 + 8	2	1	3	0.4
3	EA Model	Increasing EA	20%	20	a = 0.8, b = 2.4, c = 3.3, d =0.4	12 + 6	4	2	6	0.6
4	EA Model	Increasing EA	20%	20	a = 0.8, b = 2.4, c = 3.3, d =0.4	16 + 8	3	1	4	0.4
5	EA Model	Increasing EA	40%	20	a = 1.0, b = 2.4, c = 3.3, d =0.6	12 + 6	5	3	8	0.7
6	EA Model	Increasing EA	40%	20	a = 1.0, b = 2.4, c = 3.3, d =0.6	16 + 8	3	2	5	0.4
7	EA Model	Increasing EA	40%	20	a = 1.0, b = 2.4, c = 3.3, d =0.6	20 + 10	3	2	5	0.3
8	EA Model	Increasing EA	40%	20	a = 1.0, b = 2.4, c = 3.3, d =0.6	24 + 12	2	1	3	0.3
9	EA Model	Increasing EA	60%	20	a = 1.2, b = 2.4, c = 3.3, d =0.8	12 + 6	8	5	13	0.9
10	EA Model	Increasing EA	60%	20	a = 1.2, b = 2.4, c = 3.3, d =0.8	16 + 8	4	3	7	0.4
11	EA Model	Increasing EA	60%	20	a = 1.2, b = 2.4, c = 3.3, d =0.8	20 + 10	3	2	5	0.3
12	EA Model	Increasing EA	60%	20	a = 1.2, b = 2.4, c = 3.3, d =0.8	24 + 12	2	2	4	0.3
13	EA Model	Increasing EA	80%	20	a = 1.4, b = 2.4, c = 3.3, d =1.0	12 + 6	12	6	18	0.9
14	EA Model	Increasing EA	80%	20	a = 1.4, b = 2.4, c = 3.3, d =1.0	16 + 8	5	4	9	0.5
15	EA Model	Increasing EA	80%	20	a = 1.4, b = 2.4, c = 3.3, d =1.0	20 + 10	4	3	7	0.3
16	EA Model	Increasing EA	80%	20	a = 1.4, b = 2.4, c = 3.3, d =1.0	24 + 12	3	2	5	0.3

Additional Charts with F-15 and F-22 Losses Depicted

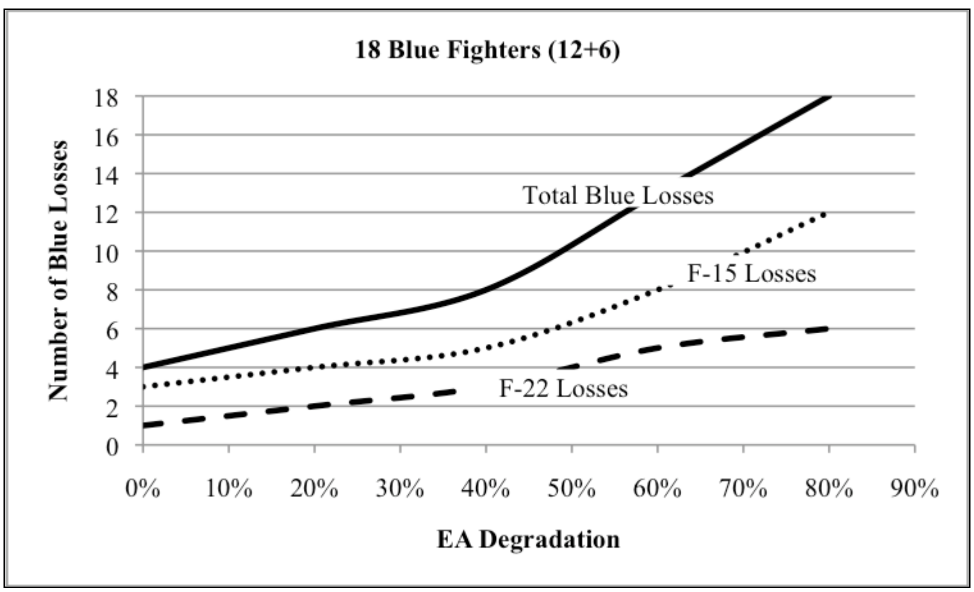

Figure 10. Blue Losses with Increasing EA for 12 F-15s and 6 F-22s

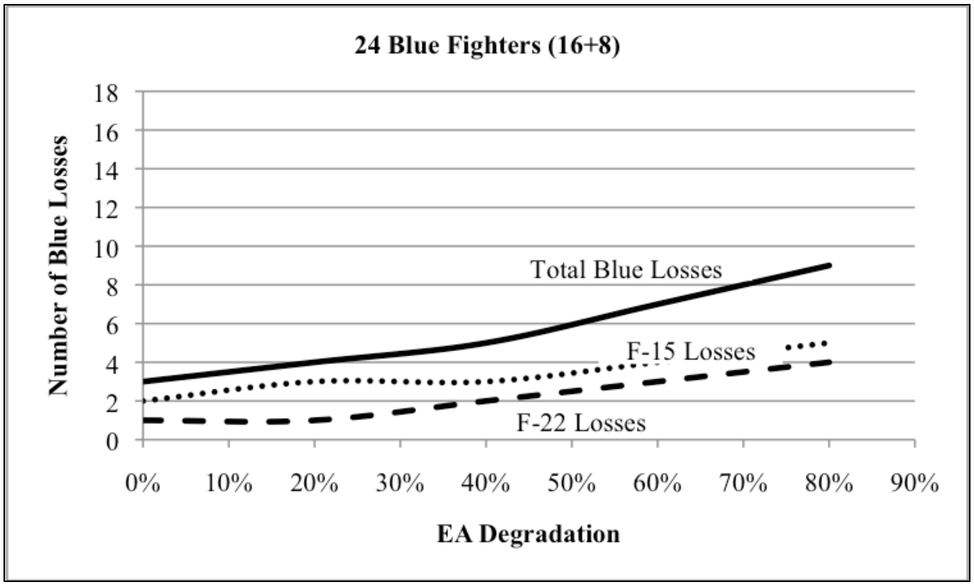

Figure 11. Blue Losses with Increasing EA for 16 F-15s and 8 F-22s

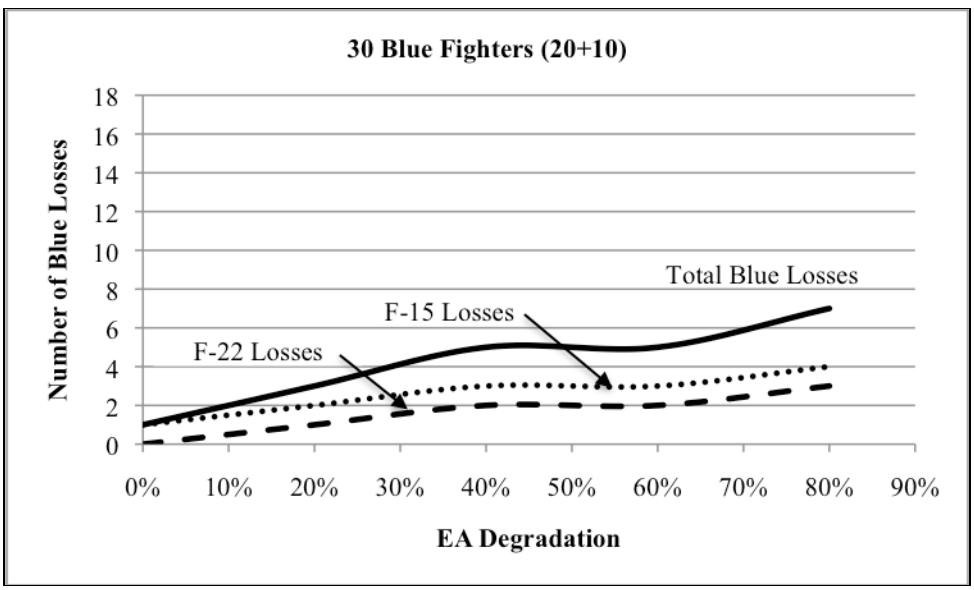

Figure 12. Blue Losses with Increasing EA for 20 F-15s and 10 F-22s

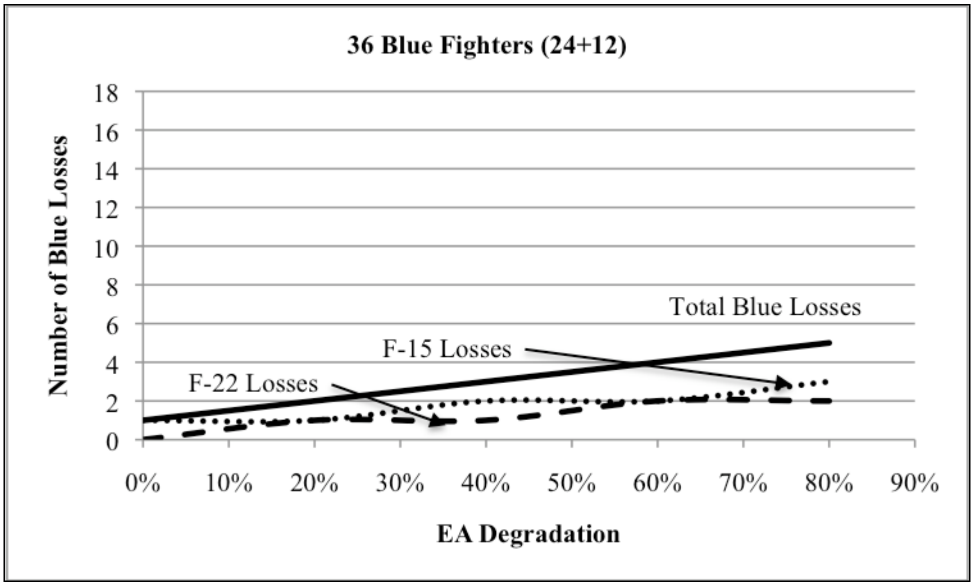

Figure 13. Blue Losses with Increasing EA for 24 F-15s and 12 F-22s

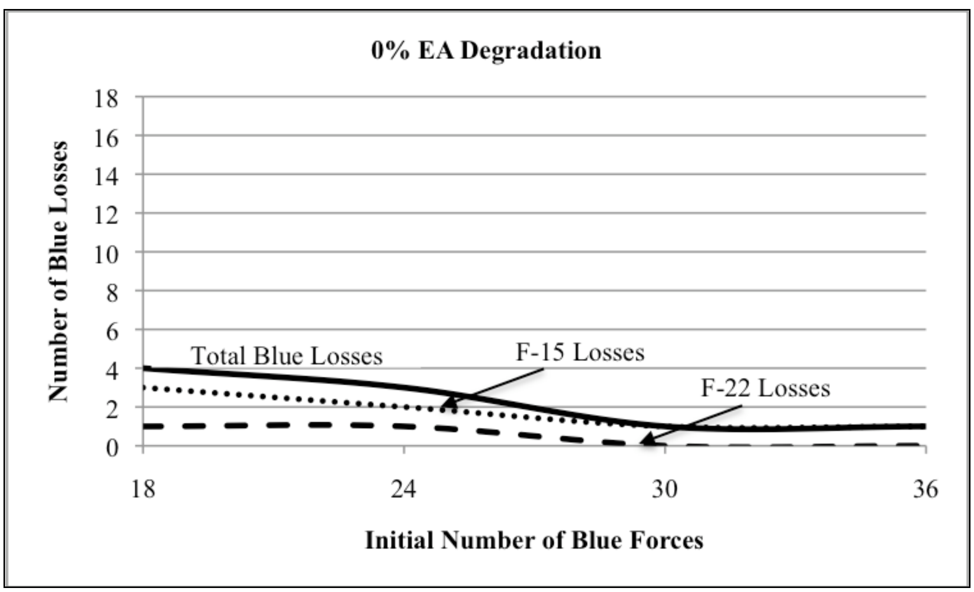

Figure 14. Blue Losses with No EA and Increasing Initial Force Strength

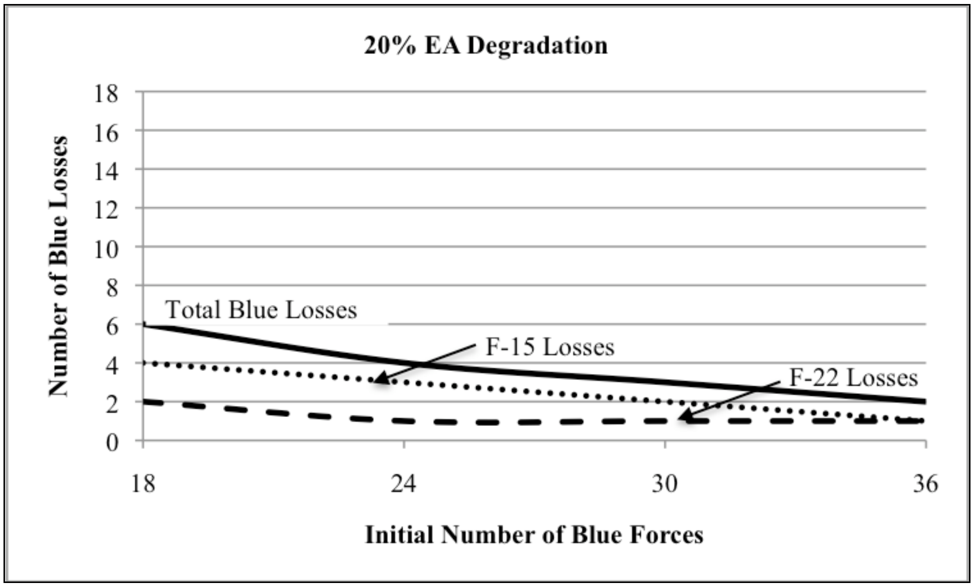

Figure 6. Blue Losses at 20% EA and Increasing Initial Force Strength

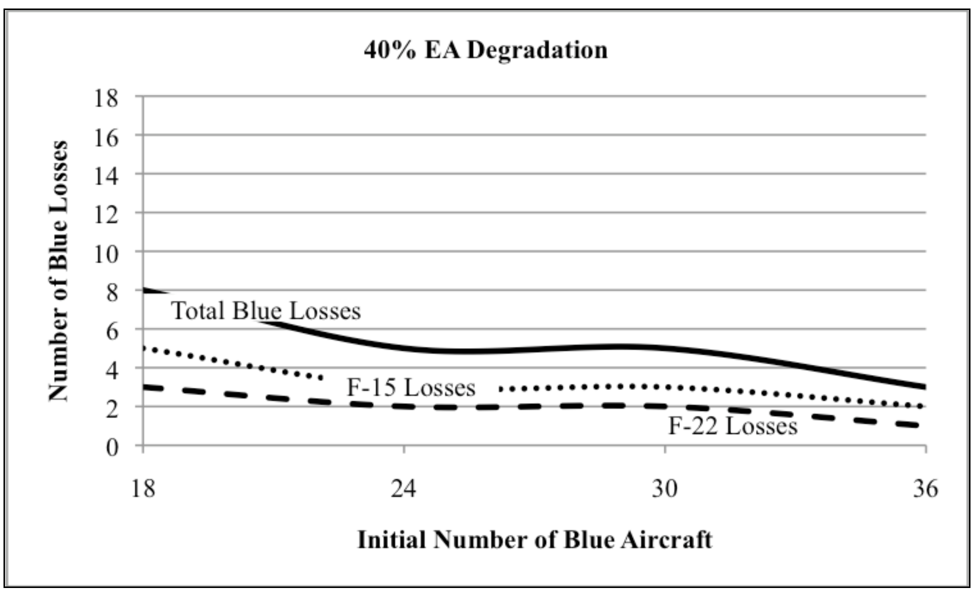

Figure 7. Blue Losses at 40% EA and Increasing Initial Force Strength

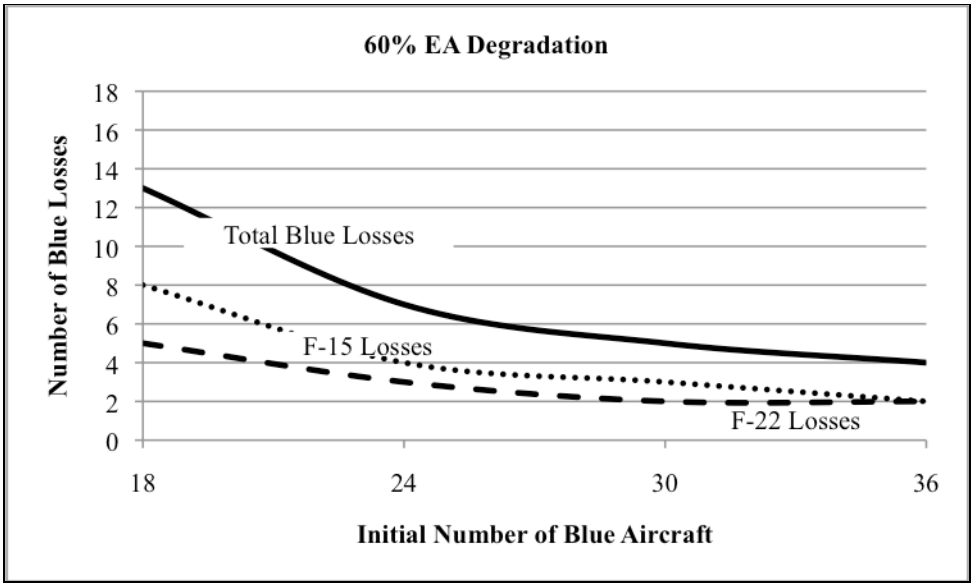

Figure 8. Blue Losses at 60% EA and Increasing Initial Force Strength

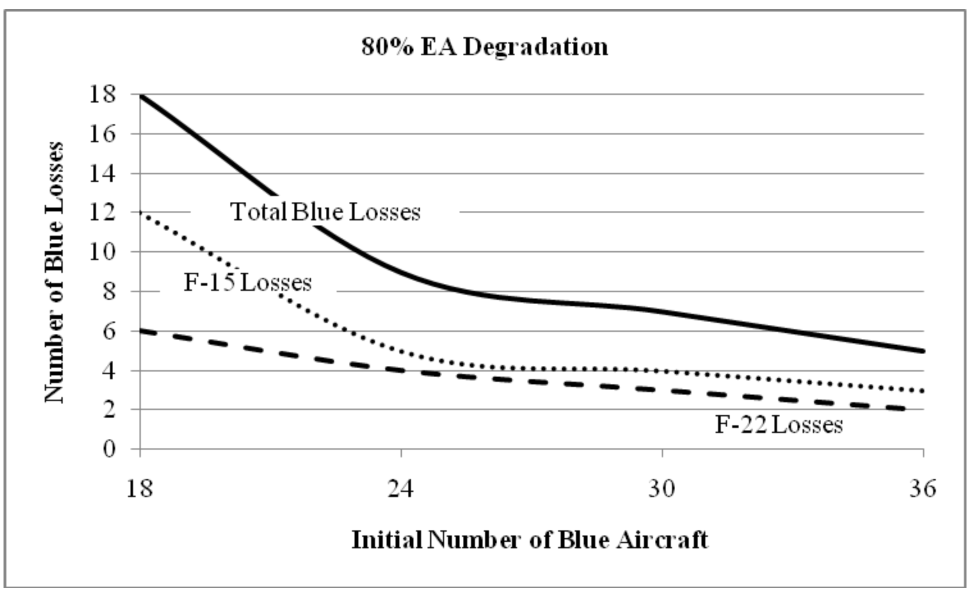

Figure 9. Blue Losses at 80% EA and Increasing Initial Force Strength

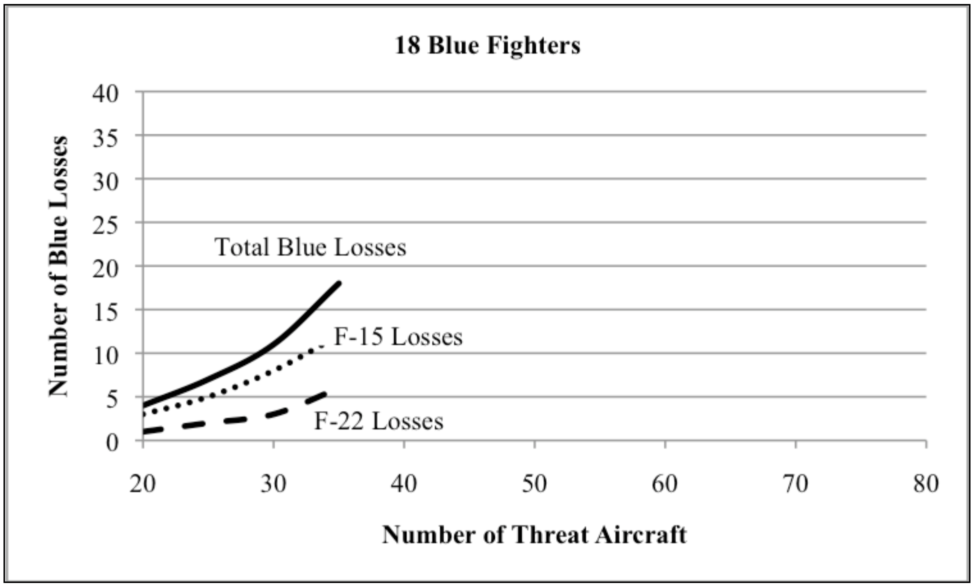

Figure 10. Blue Losses with Increasing Threat Numbers for 18 Initial Fighters

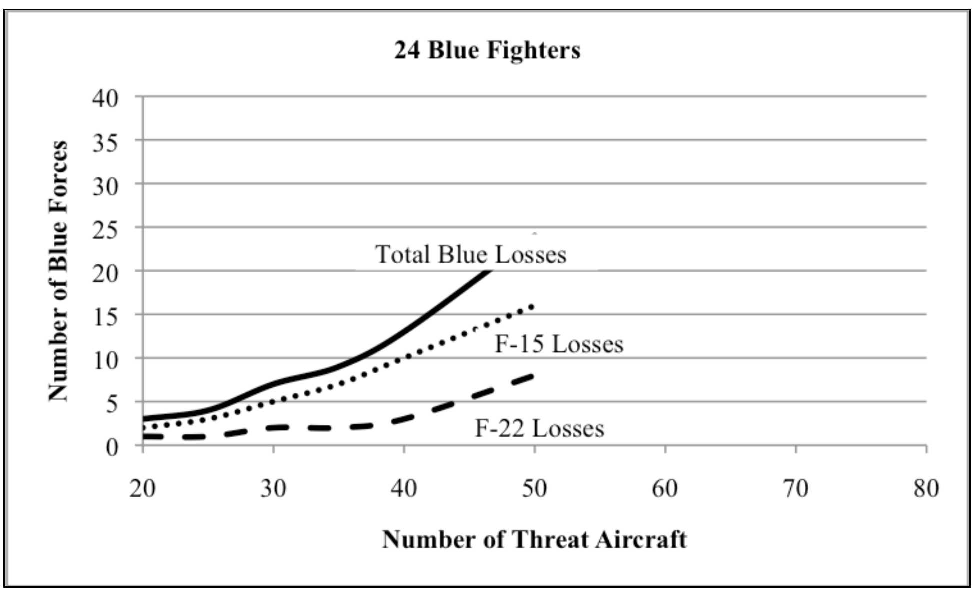

Figure 11. Blue Losses with Increasing Threat Numbers for 24 Initial Fighters

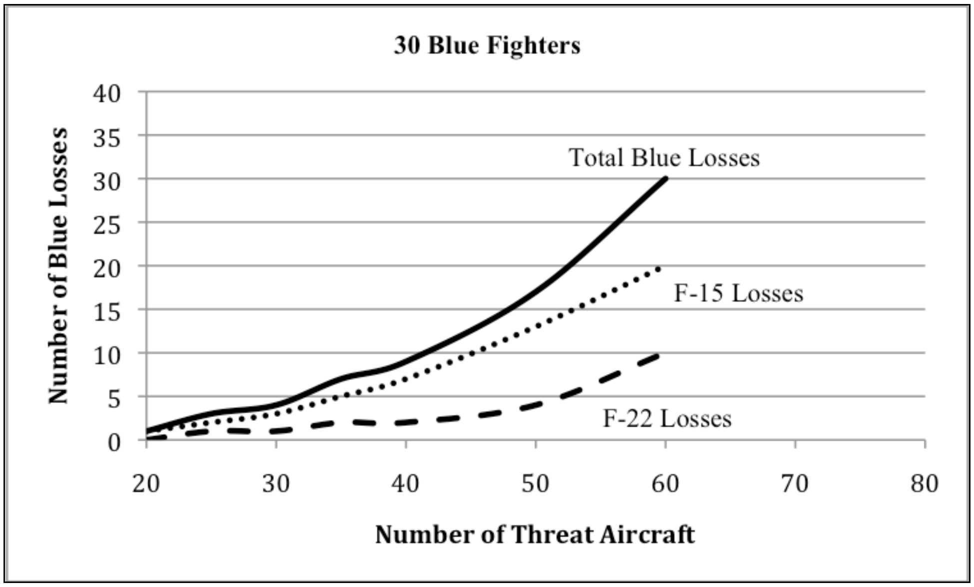

Figure 12. Blue Losses with Increasing Threat Numbers for 30 Initial Fighters

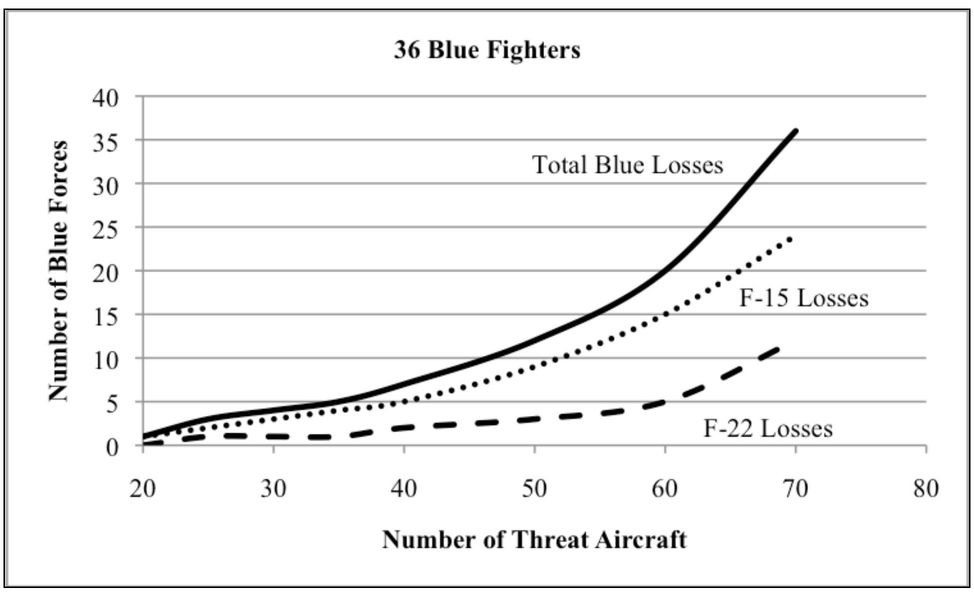

Figure 22. Blue Losses with Increasing Threat Numbers for 36 Initial Fighters

Table 18. Sample Data of Replications from Venezuela Case Study

Replication	F15 Reinforcements	F22 Reinforcements	F15 Losses	F22 Losses	Total Blue Losses	4th Gen Threat Losses	3rd Gen Threat Losses
1	3.5	2	1.5	1	2.5	31.25	11
2	3.5	1.5	1.5	0.5	2	30.5	11
3	3.25	2	1.25	1	2.25	30.5	10.5
4	3.25	2	1.25	1	2.25	30.75	10.5
5	3.25	0.5	1.25	0.5	1.75	30.25	11
6	3.5	1.5	1.5	0.5	2	30.25	10.25
7	2.25	1	1.25	1	2.25	30.5	11
8	3.25	2	1.25	1	2.25	30.25	10.5
9	3.5	2	1.5	1	2.5	30.25	11
10	3.25	2	1.25	1	2.25	30.5	11
Average	3.25	1.65	1.35	0.85	2.2	30.5	10.775
Standard Dev	0.354	0.502	0.122	0.229	0.218	0.296	0.284

BLUE DART

The Large Hole in US Power Projection

Over the past few decades, the United States military has fought comfortably under the blanket of air dominance. In the next few years, the small, almost unnoticeable hole in our blanket will grow considerably as our fighter aircraft force decreases, affecting the warfighters in the air and on the ground and bringing to question our ability to protect ourselves in support of military policies abroad.

In WWII, the benefits of air dominance took a global stage; enabling the ground forces and naval fleets to enact military might on their foes without regard. In the conflicts since, this theme has been repeated and the importance of controlling the skies has not been overlooked. In the most impressive demonstration of aerial dominance, the coalition air forces of Desert Storm, led by the US Air Force and US Navy, shut down the Iraqi ability to wage aerial warfare, guaranteeing the successful liberation of Kuwait. Victory was delivered by the large US fighter inventory capable of finding enemy aircraft, engaging them beyond visual range (BVR) and employing long range missiles, downing their enemy with unmatched success.

In 1991, the US Air Force fighter inventory numbered 4155. This number is significant for two reasons. First and foremost, a large fighter aircraft inventory allows a military the ability to maneuver aerial forces into key positions and hold them until proven otherwise. Similar to ground schemes of maneuver, aerial maneuver; the ability to intercept enemy aircraft over vast regions relies heavily on appropriately positioning aircraft in anticipation of the enemy’s attacks. Secondly, the number of fighters is in direct correlation to the number of missiles available to stop the adversary’s forces.

Without platforms capable of carrying the BVR weaponry, the ability to secure the skies may be in question. It is troubling to think of defensive forces overrun, or escorting fighters running out of munitions while protecting our strategic bombing campaign.

Today, our current fighter aircraft number 2265 and are diminishing due to increasing age and budget constraints. The lack of fighter numbers is further amplified by the massive reduction in F-22s purchased and increasing delays of the F-35. The newer, stealth fighters are multi-roled, responsible for not only air superiority but precision attack as well, shifting ordnance load outs to bombs in place of air dominance air-to-air missiles. This is significant due to the fact that all weapons are carried internally maintaining the advanced fighters' stealth signatures and every bomb carried equals less missiles carried. The table below indicates the total number of fighter aircraft available and BVR missile quantities represented by today's inventory.

Aircraft #s	1991	2011
F-4	390	---
F-15	890	588
F-16	1613	1156
F-22	---	141
# of Missiles	13352	8998/6404*

* Number of BVR missiles available if carrying bombs

These numbers may initially appear large, however; not all aircraft deploy during conflict and not all fighter aircraft launch during a single mission. This table indicates that although the new fighters are technologically advanced they are not a direct replacement for the historic numbers.

Concerns do not stop with the shortage of aircraft or missile numbers. The evolution of missile defeating technology and tactics is increasing due to its hugely cost

efficient advantages. Electronic attack (EA) targeted at fighter radars and missile seekers is inexpensive, widely proliferated and advancing in complexity daily. Unfortunately, the US is still employing the same family of missiles as it did during the Gulf War albeit in lesser numbers.

The ability to produce more air dominance fighters in the near term is not possible due to budgetary concerns and legislative pressures on defense spending. Although this would help repair the budding hole in our air dominance forces, other, more cost effective solutions should be investigated. Advancement in missile technology to ensure survivability in a dense EA environment would increase the effectiveness and produce more air-to-air victories per missile fired. Additionally, increased kinematics through advanced rocket motor technology would increase the ability for US forces to destroy their foes, regardless of their missile defeating maneuvers. Lastly, reduction in missile body size while maintaining the current missile capabilities would increase the numbers of missiles carried, restoring the posture previously accustomed to. Spending drastically less money and acquiring an advanced missile(s) capability in a considerably shortened period is the answer.

In summary, the hole in our assumed air dominance will result in a change in our military policy. With each F-15, F-16 or F-18 that flies its final flight, the US blanket of aerial comfort shrinks beyond understanding. Our ability to fight foreign wars and influence the global environment will be largely questioned whenever the opposing forces have a representative aerial component. As the hole in our air dominance blanket grows large, our ability to project power globally diminishes. Now is the time to advance

our thinking, change our approach and ensure air dominance is ours for the future generations ahead.

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Vita

Major Ronald Gilbert graduated from Norte Vista High School, Riverside, California. He attended the U.S. Air Force Academy, graduating in 1998 with a Bachelor's Degree in Aeronautical Engineering commissioned as a Second Lieutenant. He obtained a Master of Business Administration in Military Management from TUI University, California. Maj Gilbert has spent his entire career as an Air Force pilot, accumulating over 1800 flight hours, primarily in the F-15C and the F-22A. He graduated from the U.S. Air Force Weapons School, F-15C Weapons Instructor Course in 2005 and was hand-selected by the Air Force Chief of Staff to be initial cadre for the F-22A Weapons Instructor Course in 2008. Maj Gilbert now serves as a graduate student of Operational Analysis at the Air Force Institute of Technology, Wright Patterson AFB, OH. Upon graduation, he is being reassigned to Nellis AFB to serve as the 433rd Weapons Squadron, Director of Operations for F-15Cs and F-22s.

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14. ABSTRACT

Lanchester equations are used as the foundation for analysis of air superiority forces, mathematically addressing the impending shortage of the United States fighters; focusing on the role of advanced technology: stealth aircraft, air-to-air missiles, and the rapid proliferation of electronic attack capabilities. These factors are accounted for in determination of the attrition coefficients for heterogeneous fighter aircraft through a simplistic scoring methodology and compared to potential adversarial states. ARENA simulation is employed to determine minimal fighter requirements and expected blue force losses as a function of threat force size and capability.

Analysis concludes that the United States is incapable of fighting a forward deployed aerial battle against a numerically equal or superior force that employs advanced technology unless initial force strength is at least half the adversarial totals. It is recommended that the US leverage innovation and advance specific technological areas surrounding fighter force lethality and survivability to address the deficiency in aircraft numbers for the foreseeable future.

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